Quantum Mechanics & Quantum Computation

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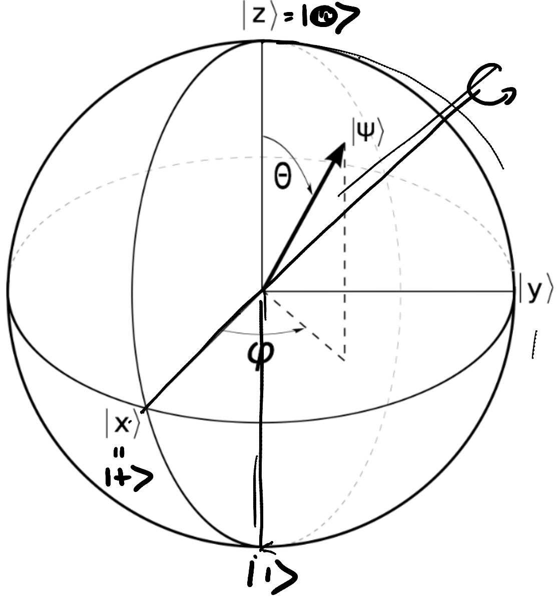
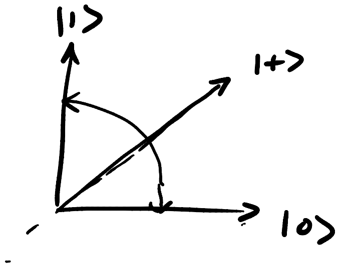
Lecture 16: Manipulating Spin

Larmor precession



$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$



g-factor = 2 electrons.
5·6 protons.

$$H = \cancel{\frac{g}{2}} \frac{e}{m} \frac{\hbar}{2} \sigma_z B_0$$ ✓

$B_0 \vec{z}$

$$\boxed{i \frac{d}{dt}|\psi(t)\rangle = H|\psi(t)\rangle}$$

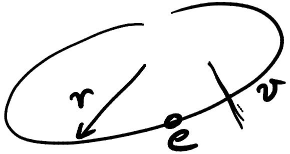
$$\sigma_z = Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\frac{\hbar}{2} \sigma_z$$

$$\omega_L = \frac{e B_0}{m}$$ Larmor frequency.

$$\Delta \phi = \omega_L \Delta t$$

$$E = -\vec{\mu} \cdot \vec{B}$$



$$\text{current} = \frac{e}{2\pi r / v} = \frac{ev}{2\pi r}$$

$$|\mu| = \text{current} \times \text{Area}$$

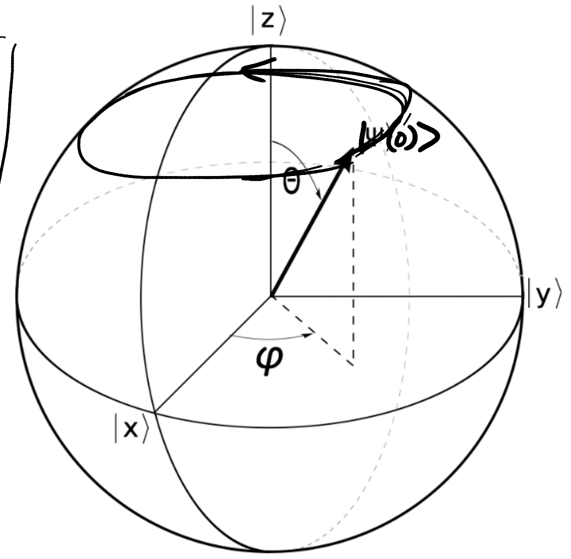
$$= \frac{ev}{2\cancel{\pi}\cancel{r}} \times \cancel{\pi} r^2$$

$$= \frac{e v r}{2} = \frac{eL}{2m}.$$

$$L = mvr \Rightarrow vr = \frac{L}{m}$$

$$\vec{\mu} = -\frac{e\vec{L}}{2m}.$$

$$E = \frac{e\vec{L}\vec{B}}{2m}.$$



$$H = \frac{e}{m}\frac{\hbar}{2}\sigma_z B_0 = \frac{e}{m}\frac{\hbar}{2} B_0 \underline{\underline{\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}}$$

$|0\rangle \quad \frac{e\hbar}{2m} B_0$

$|1\rangle \quad \frac{-e\hbar}{2m} B_0$

$$\psi(0) = \alpha|0\rangle + \beta|1\rangle. = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\frac{\theta}{2}|1\rangle.$$

$$\psi(t) = ? = e^{-i\frac{e\hbar B_0 t}{2m}/\hbar}\alpha|0\rangle + e^{i\frac{e\hbar B_0 t}{2m}/\hbar}\beta|1\rangle.$$

$$= \cancel{e^{-i\frac{eB_0 t}{2m}}}\left[\alpha|0\rangle + e^{i\frac{eB_0 t}{m}}\beta|1\rangle\right]$$

$$= \alpha|0\rangle + e^{i\omega_L t}\beta|1\rangle. \qquad \omega_L = \frac{eB_0}{m}.$$

$$= \cos\frac{\theta}{2}|0\rangle + e^{i\underline{\underline{[\phi + \omega_L t]}}}\beta|1\rangle.$$

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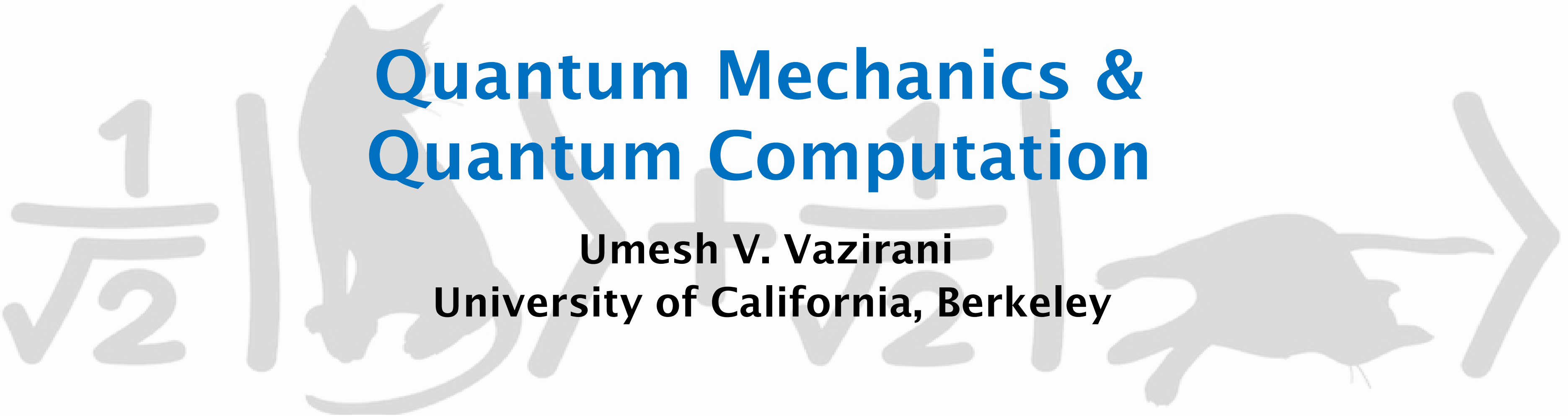

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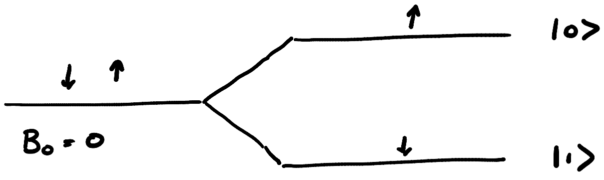
Lecture 16: Manipulating Spin

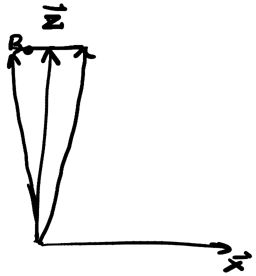
Spin Resonance

1. Turn on a large dc field $\underline{\underline{B_0}}\,\vec{z}$



$\Delta E = \hbar \omega_0 = \dfrac{\hbar e B_0}{m}$

$\omega_0 = \omega_L = \dfrac{e B_0}{m}$.



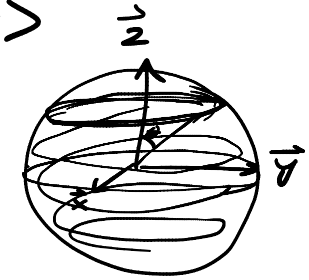
2. Turn on a small ac field $\underline{B_1}\cos \underline{\omega_0} t\,\vec{x}$

spin flips – controlled mixing between $|0\rangle$ & $|1\rangle$.

$\omega_L = \omega_0 = \dfrac{e B_0}{m}$ $\omega_R = \omega_1 = \dfrac{e B_1}{2m}$ Rabi-frequency.

$|\psi(0)\rangle = \cos\frac{\theta}{2}|0\rangle + e^{i\phi}\sin\theta/2\,|1\rangle$.

✓ $|\psi(t)\rangle = \cos\dfrac{\theta+\omega_1 t}{2}|0\rangle + e^{i(\phi+\underline{\underline{\omega_0}}t)}\sin\dfrac{\theta+\omega_1 t}{2}|1\rangle$

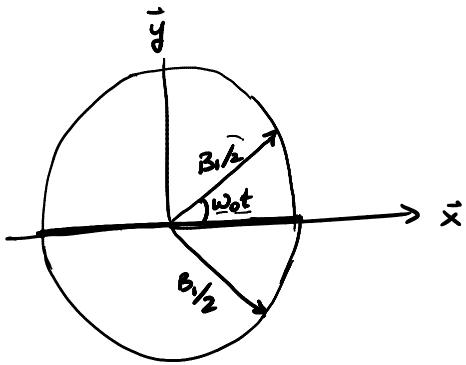


ω_0 large GHz

ω_1 small KHz

$\omega_1 \Delta t = \pi$

$B_1 \cos \omega_0 t \ \vec{x}$



$\frac{B_1}{2} \vec{x}$ effective mag field in our rotaty frame.

$$\omega_R = \omega_1 = \boxed{\frac{e \, B_1/2}{m}}$$

$$\omega_L = \omega_0 = \frac{e B_0}{m}$$

Entangled state of 2 spins.

2-particle Hamiltonian.

<u>Claim</u>: Ground state is $|\psi^-\rangle = \frac{1}{\sqrt{2}}\left[|0\rangle|1\rangle - |1\rangle|0\rangle\right]$

$$= \frac{1}{\sqrt{2}}\left[|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2\right]$$

$$H = c\,\vec{S}_1 \cdot \vec{S}_2$$

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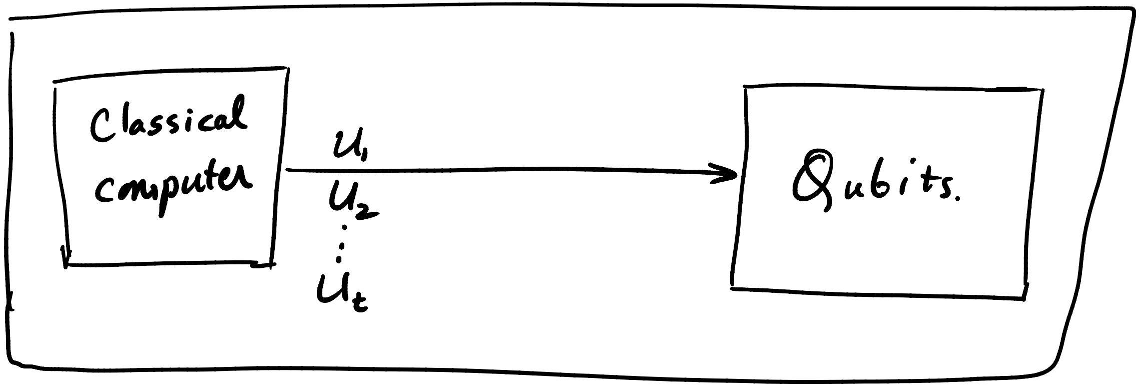
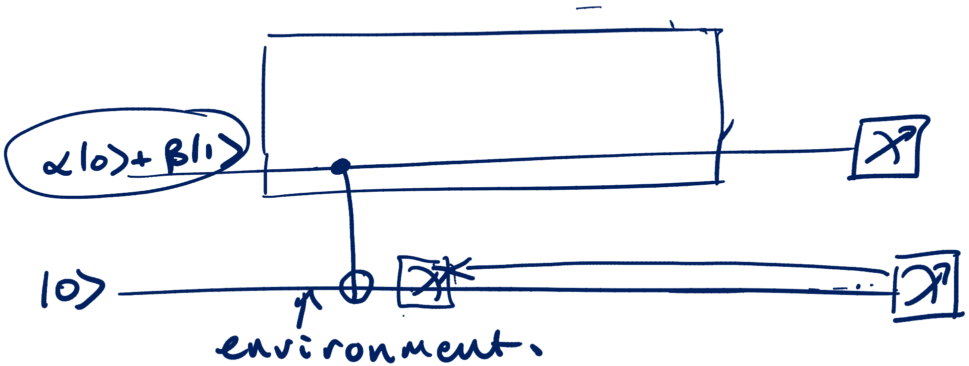
Classical control



Goal 1: control qubits by interaction.

Goal 2: isolate qubits to prevent $\underbrace{\text{inadverdent measurement}}_{\text{environmental decoherence}}$.

environmental decoherence.



Principle of deferred measurement.

0 wp $|\alpha|^2$

1 wp $|\beta|^2$

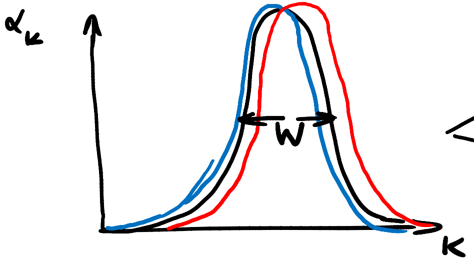
absorb photon.

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle \xrightarrow{X} \alpha|\downarrow\rangle + \beta|\uparrow\rangle$$

emit photon.

k photons → k+1 photons

k photons → k-1 photons.



$$\sum_k \alpha_k |k\rangle$$

$$\to \sum_k \alpha_k |k+1\rangle$$

$$\to \sum_k \alpha_k |k-1\rangle.$$

$$O\left(\frac{1}{W}\right)$$

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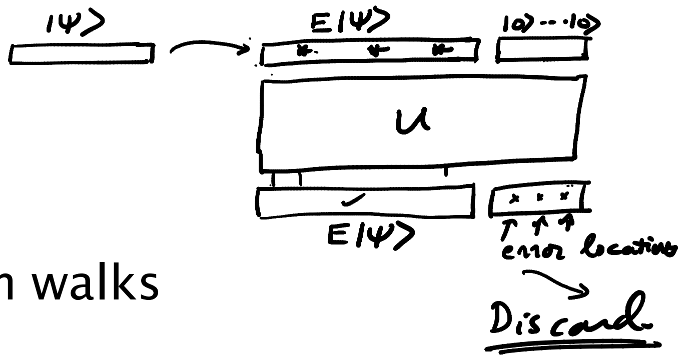
Wrapping up

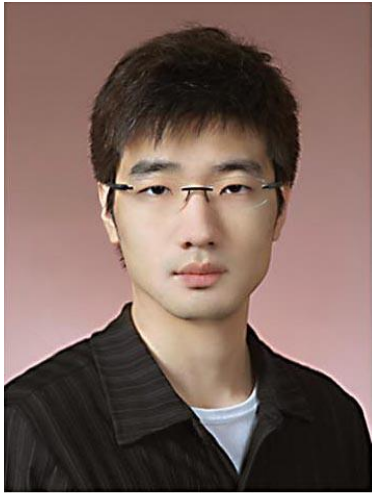
Topics we didn't cover



- Quantum error correcting codes
- Fault-tolerant quantum computation
- Algorithms – phase estimation, quantum walks
- Quantum cryptography
- Experimental realization
- Quantum Hamiltonian complexity

  - Quantum computation by teleportation
  - Certifiable quantum random numbers
  - Testing quantum computers

CHSH



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