

## Module 7 Practice problem and Homework answers

### Practice problem, page 1

*Is the research hypothesis one-tailed or two-tailed?*

Answer: one tailed

In the set up for the problem, we predicted a specific outcome – that the independent variable would directly (rather than indirectly) influence the dependent variable. This directional prediction makes this a one-tailed hypothesis.

### page 3

*What is the value of  $\Sigma x$ ?*

Answer: 71

*What is the value of  $\Sigma y$ ?*

Answer: 33

### page 4

*What is the value of  $\Sigma xy$ ?*

Answer: 235

### page 5

*What is the value of  $\Sigma x^2$ ?*

Answer: 581

Note: we calculated the answers to these four questions by summing the values of the four columns of the calculation table, which you can view in pages 3 – 5 of the practice problem. The values in column 3 were calculated by multiplying the values in columns 1 & 2 together, and the values in column 4 were calculated by squaring the values in column 1.

### page 6

*What is the value of  $b$ ?*

Answer: .247

Note: to see an explanation for how we arrived at this answer, click the “Check” button, and then the “Show Answer” button beneath this question in the practice problem page.

**Practice problems page 6, continued**

*Given the value of  $\beta$ , can you conclude that the number of friends that a child has increases as the number of friends that the parent has increases?*

Answer: yes

Because the value of  $\beta$  is positive, the relationship between the independent and dependent variables is direct, which means that as one increases, so does the other. A negative value would have indicated an indirect relationship, which would mean that as one variable increases, the other decreases.

**page 7**

*What is the value of  $\alpha$ ?*

Answer: 1.29

Note: to see an explanation for how we arrived at this answer, click the "Check" button, and then the "Show Answer" button beneath this question in the practice problem page.

*Given the value of  $\alpha$ , how many friends do you expect the child of a parent who has no close friends to have?*

Answer: 1

To answer this question, we need to construct the regression equation, using the  $\hat{y} = \alpha + \beta x$  format. In this problem, the regression equation is  $\hat{y} = 1.29 + .247x$ . To figure out the value of the dependent variable (y) when the independent variable (x) is 0, we plug 0 in for x in the equation, and solve for  $\hat{y}$ :  $\hat{y} = 1.29 + .247(0)$ ;  $\hat{y} = 1.29$ . In every case, when we plug in 0 for x, we will get  $y = \alpha$ . This is the y-intercept of the regression equation, or the point at which the line hits the Y axis.

**page 9**

*What is the predicted Y value for  $X=7$ ?*

Answer: 3.02

$$\hat{y} = 1.29 + .247(7)$$

$$\hat{y} = 3.02$$

*If a parent has 3 close friends, how many close friends can we expect the son or daughter to have?*

Answer: 2

$$\hat{y} = 1.29 + .247(3)$$

$$\hat{y} = 2$$

**Practice problem page 10**

What is the value of  $\sum(y - \bar{y})^2$

Answer: 36.38

What is the value of  $\sum(x - \bar{x})^2$

Answer: 160.92

Note: to see an explanation for how we arrived at these two answers, click the “Check” button, and then the “Show Answer” button beneath this question in the practice problem page.

**page 11**

What is the value of  $\sigma$ ?

Answer: 0.15

Note: to see an explanation for how we arrived at these two answers, click the “Check” button, and then the “Show Answer” button beneath this question in the practice problem page.

**page 12**

What is the calculated  $t$  value for this problem?

Answer: 1.647

Note: to see an explanation for how we arrived at these two answers, click the “Check” button, and then the “Show Answer” button beneath this question in the practice problem page.

**page 13**

What is the  $df$  value for this problem?

Answer: 10

$$df = N - 2; df = 12 - 2; df = 10$$

What is the critical  $t$  value for this problem?

Answer: 1.812

For a one-tailed research hypothesis and a dataset with 10 degrees of freedom, the critical  $t$  value is 1.812.

**Practice problem page 13, continued**

*Is the  $\beta$  value for this problem significantly different from 0?*

Answer: no

Because the calculated t value is not greater than the critical t value, we conclude that the  $\beta$  value is not significantly different from 0, which indicates that the relationship between the independent variable and dependent variable is not significant.

### Homework, page 1

Is the research hypothesis one-tailed or two-tailed?

Answer: two-tailed

Because the researcher is leaving open the possibility for both a positive relationship (that negative ads increase voter turnout) and a negative relationship (that negative ads decrease voter turnout), the hypothesis presented in the problem set up is two-tailed.

What is the value of  $\Sigma xy$ ?

Answer: 47,737

x (percent negative)	y (voter turnout)	x*y	x <sup>2</sup>
62	62	3844	3844
38	53	2014	1444
60	59	3540	3600
56	63	3528	3136
45	45	2025	2025
26	70	1820	676
34	72	2448	1156
31	60	1860	961
59	31	1829	3481
58	60	3480	3364
55	72	3960	3025
27	51	1377	729
27	46	1242	729
33	25	825	1089
62	66	4092	3844
27	46	1242	729
50	30	1500	2500
33	31	1023	1089
42	41	1722	1764
59	74	4366	3481
884	1057	47737	42666

### Homework page 1, continued

What is the value of  $\beta$ ?

Answer: 0.283

$$\beta = \frac{N \sum xy - \sum x \sum y}{N \sum x^2 - (\sum x)^2}$$

$$\sum xy = 47737, \sum x = 884, \sum y = 1057, \sum x^2 = 42666$$

$$\beta = \frac{20 * 47737 - 884 * 1057}{20 * 42666 - (884)^2}$$

$$\beta = \frac{954756.991 - 934388}{853320 - 781456}$$

$$\beta = \frac{20368.991}{71864}$$

$$\beta = 0.283$$

According to the value of  $\beta$ , does voter turnout increase as the percentage of negative advertisements increases? (1 = yes, 2 = no)

Answer: 1 (yes)

When the value of  $\beta$  is positive, the independent and dependent variable have a direct relationship, which means that as one increases, so does the other one.

What is the value of  $\alpha$ ?

Answer: 40.341

$$\alpha = \bar{y} - \beta \bar{x}$$

$$\sum x = 884, \sum y = 1057, \beta = 0.283$$

$$\alpha = \frac{1057}{20} - 0.283 * \frac{884}{20}$$

$$\alpha = 52.85 - 0.283 * 44.2$$

$$\alpha = 52.85 - 12.509$$

$$\alpha = 40.341$$

### Homework page 1, continued

According to this value of  $\alpha$ , what level of voter turnout should we expect for a campaign in which there are no negative advertisements?

Answer: 40.341%

The  $\alpha$  value gives us the y-intercept, which is the value of the dependent variable when the independent variable is 0.

Using the regression equation, calculate the predicted Y value for X=62.

Answer: 57.887

$$\hat{y} = \alpha + \beta x$$

$$\hat{y} = 40.341 + 0.283 * 62$$

$$\hat{y} = 40.341 + 17.546$$

$$\hat{y} = 57.887$$

What is the value of sigma (in the t-test equation)?

Answer: 0.256

$$\sigma = \frac{\sqrt{\frac{\sum(y - \hat{y})^2}{N - 2}}}{\sqrt{\sum(x - \bar{x})^2}}$$

$$\sigma = \frac{\sqrt{\frac{4238.363}{20 - 2}}}{\sqrt{3593.2}}$$

$$\sigma = \frac{\sqrt{235.465}}{\sqrt{3593.2}}$$

$$\sigma = \frac{15.345}{59.943}$$

$$\sigma = 0.256$$

Homework page 1, continued

x	y	$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
62	62	57.887	4.113	16.917	44.2	17.8	316.84
38	53	51.095	1.905	3.629	44.2	-6.2	38.44
60	59	57.321	1.679	2.819	44.2	15.8	249.64
56	63	56.189	6.811	46.390	44.2	11.8	139.24
45	45	53.076	-8.076	65.222	44.2	0.8	0.64
26	70	47.699	22.301	497.335	44.2	-18.2	331.24
34	72	49.963	22.037	485.629	44.2	-10.2	104.04
31	60	49.114	10.886	118.505	44.2	-13.2	174.24
59	31	57.038	-26.038	677.977	44.2	14.8	219.04
58	60	56.755	3.245	10.530	44.2	13.8	190.44
55	72	55.906	16.094	259.017	44.2	10.8	116.64
27	51	47.982	3.018	9.108	44.2	-17.2	295.84
27	46	47.982	-1.982	3.928	44.2	-17.2	295.84
33	25	49.68	-24.68	609.102	44.2	-11.2	125.44
62	66	57.887	8.113	65.821	44.2	17.8	316.84
27	46	47.982	-1.982	3.928	44.2	-17.2	295.84
50	30	54.491	-24.491	599.809	44.2	5.8	33.64
33	31	49.68	-18.68	348.942	44.2	-11.2	125.44
42	41	52.227	-11.227	126.046	44.2	-2.2	4.84
59	74	57.038	16.962	287.709	44.2	14.8	219.04
				4238.363			3593.20

What is the calculated  $t$  value for this problem?

Answer: 1.105

$$t = \frac{\beta}{\sigma}$$

$$\beta = 0.283, \sigma = 0.256$$

$$t = \frac{0.283}{0.256}$$

$$t = 1.105$$



**Homework page 1, continued**

*What is the critical t value for this problem?*

Answer: 2.101

The critical t value for a dataset with 20 (df = 18) cases with which we're testing a two-tailed research hypothesis is 2.101.

*Is the relationship between x and y statistically significant?*

Answer: no

Because the calculated t value (1.105) is less than the critical t value (2.101), we fail to reject the null hypothesis, and conclude that the relationship between negative ads and voter turnout is not significant.

## Homework, page 2

Is the research hypothesis one-tailed or two-tailed?

Answer: one-tailed

Because we have reason to suspect that depression decreases when time spent listening to music increases (a directional assumption), the hypothesis was written as a one-tailed test.

What is the value of  $\Sigma x \Sigma y$ ?

Answer: 762,766

$$\Sigma x \Sigma y = \Sigma x * \Sigma y = 1028 * 742 = 762,766$$

x (music time)	y (depression score)	x*y	x <sup>2</sup>
118	27	3186	13924
100	58	5800	10000
96	45	4320	9216
95	58	5510	9025
29	54	1566	841
60	62	3720	3600
65	67	4355	4225
26	64	1664	676
50	65	3250	2500
41	49	2009	1681
78	48	3744	6084
101	49	4949	10201
110	33	3630	12100
59	63	3717	3481
1028	742	51420	87554

### Homework page 2, continued

What is the value of  $\beta$ ?

Answer: -0.254

$$\beta = \frac{N \sum xy - \sum x \sum y}{N \sum x^2 - (\sum x)^2}$$

$$\sum xy = 51420, \sum x = 1028, \sum y = 742, \sum x^2 = 87554$$

$$\beta = \frac{14 * 51420 - 1028 * 742}{14 * 87554 - (1028)^2}$$

$$\beta = \frac{719880 - 762776}{1225757 - 1056784}$$

$$\beta = \frac{-42896}{168973}$$

$$\beta = -0.254$$

According to the value of  $\beta$ , does depression increase as the length of time listening to upbeat music increases? (1 = yes, 2 = no)

Answer: 2 (no)

When the value of  $\beta$  is negative, the independent and dependent variable have an indirect relationship, which means that as one increases, the other decreases.

### Homework page 2

What is the value of  $\alpha$ ?

Answer: 71.651

$$\alpha = \bar{y} - \beta \bar{x}$$

$$\sum x = 1028, \sum y = 742, \beta = -0.254$$

$$\alpha = \frac{742}{14} - (-0.254) * \frac{1028}{14}$$

$$\alpha = 53.00 - (-0.254) * 73.43$$

$$\alpha = 53.00 - (-18.651)$$

$$\alpha = 71.651$$

### Homework page 2, continued

According to this value of  $\alpha$ , if a patient has a depression scale score of 71.5, about how many minutes of upbeat music do you expect that he is listening to each day?

Answer: 0

The alpha value gives us the predicted value of the dependent variable (in this case, depression scale score) when the independent variable (music time) is 0. When the predicted score on the dependent variable and alpha have the same value, then the value of the independent variable must be 0. Confirm this by looking at the regression equation and plugging in 71.651 (or 71.65) for  $\alpha$  and 0 for  $x$ .

Using the regression equation, calculate the predicted  $Y$  value for  $X=118$ .

Answer: 41.7

$$\hat{y} = \alpha + \beta x$$

$$\hat{y} = 71.651 + (-0.254) * 118$$

$$\hat{y} = 71.651 - 29.972$$

$$\hat{y} = 41.7$$

What is the value of sigma (in the t-test equation)?

Answer: 0.088

$$\sigma = \frac{\sqrt{\frac{\sum(y - \hat{y})^2}{N - 2}}}{\sqrt{\sum(x - \bar{x})^2}}$$

$$\sigma = \frac{\sqrt{\frac{1112.149}{14 - 2}}}{\sqrt{12069.428}}$$

$$\sigma = \frac{\sqrt{92.679}}{\sqrt{12069.428}}$$

$$\sigma = \frac{9.627}{109.861}$$

$$\sigma = 0.088$$

Homework page 2, continued

x	y	$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
118	27	41.685	-14.685	215.649	73.429	44.571	1986.574
100	58	46.254	11.746	137.969	73.429	22.571	706.018
96	45	47.270	-2.270	5.153	73.429	21.571	509.45
95	58	47.524	10.476	109.747	73.429	-44.429	465.308
29	54	64.279	-10.279	105.658	73.429	-13.429	1973.936
60	62	56.409	5.591	31.259	73.429	-8.429	180.338
65	67	55.140	11.860	140.660	73.429	-47.429	71.048
26	64	65.040	-1.040	1.082	73.429	-23.429	2249.51
50	65	58.948	6.052	36.627	73.429	-32.429	548.918
41	49	61.232	-12.232	149.622	73.429	4.571	1051.64
78	48	51.839	-3.839	14.738	73.429	27.571	20.894
101	49	46.001	2.999	8.994	73.429	36.571	760.16
110	33	43.716	-10.716	114.833	73.429	-14.429	1337.438
59	63	56.663	6.337	40.158	73.429	26.571	208.196
				1112.149			12069.428

What is the calculated  $t$  value for this problem?

Answer: -2.887

$$t = \frac{\beta}{\sigma}$$

$$\beta = -0.254, \sigma = 0.088$$

$$t = \frac{-0.254}{0.088}$$

$$t = -2.887$$

What is the critical  $t$  value for this problem?

Answer: 1.782

The critical  $t$  value for a dataset with 14 (df = 12) cases with which we're testing a one-tailed research hypothesis is 1.782.

### Homework page 2, continued

*Is the relationship between x and y statistically significant?*

Answer: yes

Because the magnitude of the calculated t value (-2.887) is greater than the critical t value (1.782), we reject the null hypothesis, and conclude that the relationship between the independent and dependent variables is significant.

### Homework, page 3

*Is the research hypothesis one-tailed or two-tailed?*

Answer: two-tailed

The problem set-up leaves open the possibility that the size of the audience could have either a negative or a positive effect on the team's score, which makes this a two-tailed hypothesis.

*What is the value of  $\Sigma x^2$ ?*

Answer: 7432

To find this value, square each x value, and then sum those squares. (See the table below).

x (crowd size)	y (points scored)	x*y	x <sup>2</sup>
35	58	2030	1225
31	41	1271	961
29	46	1334	841
11	42	462	121
11	48	528	121
13	43	559	169
30	56	1680	900
25	40	1000	625
36	69	2484	1296
20	57	1140	400
17	57	969	289
22	65	1430	484
280	622	14887	7432

### Homework page 4, continued

What is the value of  $\beta$ ?

Answer: 0.416

$$\beta = \frac{N \sum xy - \sum x \sum y}{N \sum x^2 - (\sum x)^2}$$

$$\sum xy = 14887, \sum x = 280, \sum y = 622, \sum x^2 = 7432$$

$$\beta = \frac{12 * 14887 - 280 * 622}{12 * 7432 - (280)^2}$$

$$\beta = \frac{178644 - 174160}{89184 - 78400}$$

$$\beta = \frac{4484}{10784}$$

$$\beta = 0.416$$

Points earned by the home team \_\_\_\_\_ as the team crowd creases. (1 = increases, 2 = decreases)

Answer: 1 (increases)

When the value of  $\beta$  is positive, the dependent and independent variables either increase together or decrease together.

What is the value of  $\alpha$ ?

Answer: 42.126

$$\alpha = \bar{y} - \beta \bar{x}$$

$$\sum x = 280, \sum y = 622, \beta = 0.416$$

$$\alpha = \frac{622}{12} - 0.416 * \frac{280}{12}$$

$$\alpha = 51.833 - (0.416 * 23.333)$$

$$\alpha = 51.833 - 9.707$$

$$\alpha = 42.126$$

### Homework page 3, continued

How many points would you expect the home team to score if there were no home-team fans in attendance?

Answer: 42

The independent variable in this problem is home-team attendance. If this value is 0, then the value of the dependent variable (number of points scored) is the same as  $\alpha$ , which is the y-intercept, or the point on the regression line at which  $x=0$ .

Using the regression equation, calculate the predicted Y value for  $X=35$

Answer: 57

$$\hat{y} = \alpha + \beta x$$

$$\hat{y} = 42.126 + (0.416) * 35$$

$$\hat{y} = 42.126 + 14.560$$

$$\hat{y} = 57 \text{ (rounded to a whole number)}$$

What is the value of sigma (in the t-test equation)?

Answer: 0.317

$$\sigma = \frac{\sqrt{\frac{\sum(y - \hat{y})^2}{N - 2}}}{\sqrt{\sum(x - \bar{x})^2}}$$

$$\sigma = \frac{\sqrt{\frac{902.296}{12 - 2}}}{\sqrt{898.668}}$$

$$\sigma = \frac{\sqrt{90.230}}{\sqrt{898.668}}$$

$$\sigma = \frac{9.499}{29.978}$$

$$\sigma = 0.317$$



Homework page 3, continued

x	y	$\hat{y}$	$y-\hat{y}$	$(y-\hat{y})^2$	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
35	58	56.687	1.313	1.724	23.333	11.667	136.119
31	41	55.023	-14.023	196.645	23.333	7.667	58.783
29	46	54.191	-8.191	67.092	23.333	5.667	32.115
11	42	46.703	-4.703	22.118	23.333	-12.333	152.103
11	48	46.703	1.297	1.682	23.333	-12.333	152.103
13	43	47.535	-4.535	20.566	23.333	-10.333	106.771
30	56	54.607	1.393	1.940	23.333	6.667	44.449
25	40	52.527	-12.527	156.926	23.333	1.667	2.779
36	69	57.103	11.897	141.539	23.333	12.667	160.453
20	57	50.447	6.553	42.942	23.333	-3.333	11.109
17	57	49.199	7.801	60.856	23.333	-6.333	40.107
22	65	51.279	13.721	188.266	23.333	-1.333	1.777
				902.296			898.668

What is the calculated t value for this problem?

Answer: 1.312

$$t = \frac{\beta}{\sigma}$$

$$\beta = 0.416, \sigma = 0.317$$

$$t = \frac{0.416}{0.317}$$

$$t = 1.312$$

What is the critical t value for this problem?

Answer: 2.228

The critical t value for a dataset with 12 (df = 10) cases with which we're testing a two-tailed research hypothesis is 2.228.

**Homework page 3, continued**

*Is the relationship between  $x$  and  $y$  statistically significant?*

Answer: no

Because the calculated  $t$  value (1.312) is not greater than the critical  $t$  value (2.228), we fail to reject the null hypothesis, and conclude that the relationship between the independent and dependent variables is not significant.