Chapter 1 Overview

• Problem solving
• Data structures and abstract data types
• The properties and categories of algorithms
• Evaluating the efficiency of the algorithms
1.1 Problem solving

- Goal of writing computer programs?
  - To solve practical problems
- Problem Abstraction
  - Analyze requirements and build a problem model
- Data Abstraction
  - Determine an appropriate data structure to represent a certain mathematical model
- Algorithm Abstraction
  - Design suitable algorithms for the data model
- Data structures + Algorithms \( \Rightarrow \) Programs
  - Simulate and solve practical problems
1.1 Problem solving

Farmer Crosses River Puzzle

cabbage

sheep

wolf
1.1 Problem solving

- **Problem abstraction**: FSWC crossing over the river
  - Only the farmer can row the boat
  - There are only two seats on the boat including the farmer
  - “Wolf and sheep”, “sheep and cabbages” cannot stay alone without the accompany of the farmer

- **Data abstraction**: graph model
  - Unreasonable state: WS, FC, SC, FW, WSC, F
  - The vertex represents the “original bank status” (10 states, including “empty”)
  - edge: state transition as the result of a reasonable operation (cross over the river)

Farmer Crosses River Puzzle

Farmer is abbreviated as F
Sheep is abbreviated as S
Wolf is abbreviated as W
Cabbage is abbreviated as C
1.1 Problem solving

Farmer Crosses River Puzzle

- Data structure
  - Adjacency matrix

- Algorithm abstraction:
  - The shortest path

Farmer is abbreviated as F
Sheep is abbreviated as S
Wolf is abbreviated as W
cabbage is abbreviated as C
Questions: process of problem solving

• Farmer Crosses River Puzzle —— The shortest path model
  • Problem abstraction?
  • Data abstraction?
  • Algorithm abstraction?
  • You may write programs to achieve it.

• Any other model?
Chapter 1 Overview

- Problem solving
- **Data structures and abstract data types**
- The properties and categories of algorithms
- Evaluating the efficiency of the algorithms
1.2 What is data structure

- **Structure**: entity + relation
- **Data structure**:
  - Data organized according to logical relationship
  - Stored in computer according to a certain storage method
  - A set of operations are defined on these data
1.2 What is data structure

Logical organization of data structure

- **Linear Structure**
  - Linear lists (list, stack, queue, string, etc.)

- **Nonlinear Structure**
  - Trees (binary tree, Huffman tree, binary search tree, etc.)
  - Graphs (directed graph, undirected graph, etc.)

- **Graph ⊇ tree ⊇ binary tree ⊇ linear list**
1.2 What is data structure

Storage structure of data

- **Mapping** from logical structure to the physical storage space

**Main memory (RAM)**

- Coded in non-negative integer address, set of adjacent unit
- The basic unit is the byte
- The time required to access different addresses are basically the same (random access)
1.2 What is data structure

Storage structure of data

- For logical structure \((K, r)\), in which \(r \in \mathbb{R}\)
  - For the node set \(K\), establish a mapping from \(K\) to \(M\)
    memory unit: \(K \rightarrow M\), for every node \(j \in K\), it corresponds to a unique continuous storage area \(C\) in \(M\)

```
int a[3]
a[0] a[1] a[2]
```

---

Storage mapping

```
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</table>
```

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Main memory
### 1.2 What is data structure

#### Storage structure of data

- **Relation tuple** $(j_1, j_2) \in r$
  
  $(j_1, j_2 \in K$ are nodes $)$
  
  - Sequence: storage units of data are **adjacent**

  ![Sequence example](image)

- **Link**: a pointer points to the storage address, referring to a certain connection

  ![Link example](image)

- **Four kinds**: **Sequence, link, index, hash**
1.2 What is data structure

Abstract Data Type

- Abbreviated as **ADT** (Abstract Data Type)
  - A set of operations built upon a mathematical model
  - Has nothing to do with the physical storage structure
  - The software system is built upon the data model (object oriented)

- The development of **Modularization**
  - Hide the details of the implementation and operations of the internal data structures
  - Software reuse
1.2 What is data structure

ADT do not care about storage details

— for example, brackets matching algorithm of C++ version

```cpp
void BracketMatch(char *str) {
    Stack<char> S; int i; char ch;
    // The stack can be sequential
    // or linked, both are referenced
    // in the same way
    for(i=0; str[i]!="\0"; i++) {
        switch(str[i]) {
            case '(': case '[': case '{':
                S.Push(str[i]); break;
            case ')': case ']': case '}
                if (S.IsEmpty()) {
                    cout<<"Right brackets excess!";
                    return;
                } else {
                    ch = S.GetTop();
                    if (Match(ch,str[i]))
                        ch = S.Pop();
                    else {
                        cout<<"Brackets do not match!";
                        return;
                    }
                }
        } /*else*/
    } /*switch*/
} /*for*/
if (S.IsEmpty())
    cout<<"Brackets match!"
else cout<<"Left brackets excess!";
```
1.2 What is data structure
Sequential stack brackets matching algorithm of C version
(different from the linked stack)

```c
void BracketMatch(char *str) {
    SeqStack S; int i; char ch;
    InitStack(&S);
    for(i=0; str[i]!='\0'; i++) {
        switch(str[i]) {
            case '(': case '[': case '{':
                Push(&S,str[i]); break;
            case ')': case ']': case '}
                if (IsEmpty(&S)) {
                    printf("\nRight brackets excess!");
                    return;
                } else {
                    GetTop (&S,&ch);
                    if (Match(ch,str[i]))
                        Pop(&S,&ch);
                    else {
                        printf("\nBrackets don’t match!");
                        return;
                    }
                } /*else*/
            } /*switch*/
        } /*for*/
        if (IsEmpty(&S))
            printf("\nBrackets match!");
        else printf("\nLeft brackets excess");
    } /*else*/
}
```
1.2 What is data structure
Linked stack brackets matching algorithm of C version
(different from the sequential stack)

void BracketMatch(char *str) {
    LinkStack S; int i; char ch;
    InitStack(&S);
    for(i=0; str[i]!='$'0'; i++) {
        switch(str[i]) {
            case '(' : case '[' : case '{' :
                Push(&S, str[i]);
                break;
            case ')' : case ']' : case '}' :
                if (IsEmpty(S)) {
                    printf("\nRight brackets excess!");
                    return;
                } else {
                    GetTop(&S,&ch);
                    if (Match(ch,str[i]))
                        Pop(&S,&ch);
                    else {
                        printf("\nBrackets don't match!");
                        return;
                    } /*else*/
                } /*switch*/
            /*for*/
        } /*else*/
    } /*for*/
    if (IsEmpty(S))
        printf("\nBrackets match!");
    else printf("\nLeft brackets excess");
}
1.2 What is data structure

Abstract Data Type

• Two-tuples of abstract data structure
  \(<\text{Data object } D, \text{ data operation } P>\)
• Firstly, defines logical structure; then data operations
  • **Logical structure**: relationship between data objects
  • **Operations**: algorithms running on the data
1.2 What is data structure

Example: abstract data type of stack

- Logical structure: linear list
- Operation: Restricted access
  - Only allow for the insert, delete operation at the top of the stack
  - push, pop, top, isEmpty

```cpp
template <class T> // Element type of stack is T
class Stack {
public: // Stack operation set
  void clear(); // Turned into an empty stack
  bool push(const T item); // Push item into the stack, return true if succeed, otherwise false
  bool pop(T & item); // Pop item out of the stack, return true if succeed, otherwise false
  bool top(T & item); // Read item at the top of the stack, return true if succeed, otherwise false
  bool isEmpty(); // If the stack is empty return true
  bool isFull(); // If the stack is full return true
};
```
1.2 What is data structure

Questions about abstract data type

• How to present a logical structure in an ADT?

• Is abstract data type equivalent to the class definition?

• Can you define a ADT without templates?
Chapter 1 Overview

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1.3 Algorithm

Problem—Algorithm—Program

Goal: problem solving

- **Problem** (a function)
  - A mapping from input to output.

- **Algorithm** (a method)
  - The description for specific problem solving process is a finite sequence of instructions

- **Program**
  - It is the algorithm implemented using a computer programming language.
1.3 Algorithm

The properties of algorithms

• Generality
  - Solve problems with parametric input
  - Ensure the correctness of the computation results

• Effectiveness
  - Algorithm is a sequence of finite instructions
  - It is made up of a series of concrete steps

• Certainty
  - In the algorithm description, which step will to be performed must be clear

• Finiteness
  - The execution of the algorithm must be ended in a finite number of steps
  - In other words, the algorithm cannot contain an endless loop
1.3 Algorithm

Queen problem (Four Queens)

- **Solution** \( <x_1, x_2, x_3, x_4> \) (Place the column number)
- **Search space**: quadtrees
1.3 Algorithm

Basic classification of algorithms

- **Enumeration**
  - Sequential search for value K

- **Backtracking, search**
  - Eight queens problem, traversal of trees and graphs

- **A recursive divide and conquer**
  - Binary search, quick sort, merge sort

- **Greedy**
  - Huffman coding tree, Dijkstra algorithm for shortest path, Prim algorithm for minimum spanning tree

- **Dynamic programming**
  - Floyd algorithm for shortest path
1.3 Algorithm

Sequential Search

template <class Type>
class Item {
private:
    Type key;        // the key field
    // other fields
public:
    Item(Type value):key(value) {}          // get the key
    Type getKey() {return key;}             // set the key
    void setKey(Type k){ key=k;}
};
vector<Item<Type>*>* dataList;
template <class Type> int SeqSearch(vector<Item<Type>*>*& dataList, int length, Type k) {
    int i=length;
    dataList[0]->setKey (k);                // the zero-th element is a sentinel
    while(dataList[i]->getKey()! =k) i--;    // return the position of the element
    return i;
}
1.3 Algorithm

Binary search

For sequential linear list that is in order

- \( k_{mid} \): The value of the element that is in the middle of the array
  - If \( k_{mid} = k \), the search is successful
  - If \( k_{mid} > k \), continue searching in the left half
  - Otherwise, if \( k_{mid} < k \), You can ignore the part that before mid and search will go on in the right part

- **Fast**
  - \( k_{mid} = k \), the search ends up successfully
  - \( k_{mid} \neq k \), reduce half of the searching range at least
1.3 Algorithm

Use binary search to find value K

```cpp
template <class Type> int BinSearch (vector<Item<Type>*>* &dataList, int length, Type k) {
    int low=1, high=length, mid;
    while (low<=high) {
        mid=(low+high)/2;
        if (k<dataList[mid]->getKey())
            high = mid-1; // decrease the upper bound of the search interval
        else if (k>dataList[mid]->getKey())
            low = mid+1; // decrease the lower bound of the search interval
        else return mid; // find value K and return the position
    }
    return 0; // fail to search and return 0
}
```
1.3 Algorithm

Illustration for binary search

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>15</td>
<td>17</td>
<td>18</td>
<td>22</td>
<td>35</td>
<td>51</td>
<td>60</td>
<td>88</td>
<td>93</td>
</tr>
</tbody>
</table>

Search the key value 18  low=1  high=9  K=18

the first time : mid=5; array[5]=35>18
    high=4; (low=1)
the second time : mid=2; array[2]=17<18
    low=3; (high=4)
the third time : mid=3; array[3]=18=18
    mid=3 ; return 3
Question: The time and space restrictions for algorithms
Design an algorithm that moves the elements of the array A(0..n-1) to
the right place by k positions circularly. The original array is supposed
to be a_0, a_1, ..., a_{n-2}, a_{n-1}; the array that has been moved will be a_{n-k}, a_{n-k+1}, ..., a_0, a_1, ..., a_{n-k-1}. You are required to just use an extra space that
is equivalent to an element, and the total number of moving and
exchanging is only linearly correlated with n. For example, n=10, k=3
The original array: 0 1 2 3 4 5 6 7 8 9
The final array: 7 8 9 0 1 2 3 4 5 6
Chapter 1 Overview

- Problem solving
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- Evaluating the efficiency of the algorithms
Asymptotic analysis of algorithm

\[ f(n) = n^2 + 100n + \log_{10} n + 1000 \]

- \( f(n) \) is the growth rate as the data scale of \( n \) gradually increases.
- When \( n \) increases to a certain value, the item with the highest power of \( n \) in the equation has the biggest impact.
  - Other items can be neglected.
Asymptotic analysis of algorithm: Big O notation

- The definition domain of function \( f \) and \( g \) is nature numbers, the range is non negative real numbers.

- **Definition**: If positive number \( c \) and \( n_0 \) exists, which makes for any \( n \geq n_0 \), \( f(n) \leq cg(n) \),

- Then \( f(n) \) is said to be in the set of \( O(g(n)) \), abbreviated as \( f(n) = O(g(n)) \)

- Big O notation: it represents the upper bound of the growth rare of a function
  - There could be more than one upper bounds of the growth rare of a function

- When the upper bound and the lower bound are the same, you can use Big \( \Theta \) notation.
1.4 Complexity analysis of algorithm

**Big O notation**

- \( f(n) = O(g(n)) \), only when
  - There exists two parameters \( c > 0 \), \( n_0 > 0 \), for any \( n \geq n_0 \), \( f(n) \leq cg(n) \)
- iff \( \exists c, n_0 > 0 \) s.t. \( \forall n \geq n_0 : 0 \leq f(n) \leq cg(n) \)

\[ n \text{ is large enough} \]
\[ g(n) \text{ is the upper bound of } f(n) \]
1.4 Complexity analysis of algorithm

**Time unit of Big O notation**

- Simple boolean or arithmetic operations
- Simple I/O
  - Input or output of a function
    For example, operations such as read data from an array
  - Files I/O operations or keyboard input are not excluded
- Return of function
1.4 Complexity analysis of algorithm

Rules of operation of Big O notation

- **Rule of addition:** $f_1(n) + f_2(n) = \mathcal{O}(\max(f_1(n), f_2(n)))$
  - Sequential structure, if structure, switch structure

- **Rule of Multiplication:** $f_1(n) \cdot f_2(n) = \mathcal{O}(f_1(n) \cdot f_2(n))$
  - for, while, do-while structure

\[
\sum_{i=0}^{n-1} (n - i) = \frac{n(n - 1)}{2} = \frac{n^2 - n}{2} = \mathcal{O}(n^2)
\]
Asymptotic analysis of algorithm: Big $\Omega$ notation

- If positive number $c$ and $n_0$ exists, which makes for any $n \geq n_0$, $f(n) \geq cg(n)$,
- Then $f(n)$ is said to be in the set of $O(g(n))$, abbreviated as $f(n) = O(g(n))$, or $f(n) = O(g(n))$
- The only difference of Big O notation and Big $\Omega$ notation is the direction of inequation.
- When you adopt the $\Omega$ notation, you’d better find the tightest (largest) lower bound of all the lower bound of the growth rate of the function.
1.4 Complexity analysis of algorithm

**Big Ω notation**

- \( f(n) = \Omega(g(n)) \)
  - iff \( \exists c, n_0 > 0 \) s.t. \( \forall n \geq n_0, 0 \leq cg(n) \leq f(n) \)

- The only difference with Big O notation is the direction of inequation

\[ \text{n is large enough} \]
\[ g(n) \text{ is the lower bound of } f(n) \]
1.4 Complexity analysis of algorithm

Asymptotic analysis of algorithm: Big $\Theta$ notation

- When the upper bound and the lower bound are the same, you can use $\Theta$ notation.

- Definition:
  If a function is in the set of $O(g(n))$ and $\Omega(g(n))$, it is called $\Theta(g(n))$.

- In other words, when the upper bound and the lower bound are the same, you can use Big $\Theta$ notation.

- There exist $c_1$, $c_2$, and positive integer $n_0$, which makes for any positive integer $n > n_0$, The following two inequality are correct at the same time:

$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$$
1.4 Complexity analysis of algorithm

**Big $\Theta$ notation**

- $f(n) = \Theta(g(n))$
  
  - iff $\exists c_1, c_2, n_0 > 0$ s.t. $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$, $\forall n \geq n_0$

- When the upper bound and the lower bound are the same, you can use $\Theta$ notation.

$n$ is large enough
g(n) has the same growth rate with $f(n)$
1.4 Complexity analysis of algorithm

The growth rate curve of function $f(n)$

- $2^n$
- $n^2$
- $n \log_2 n$
- $n$
- $\log_2 n$
1.4 Complexity analysis of algorithm

Problem space vs time overhead

The input data space of problem

The diagram shows the relationship between time cost and scale n for problem space vs time overhead. The graph includes lines for worst, average, and best cases, illustrating how time cost varies with the scale of the problem.
1.4 Complexity analysis of algorithm

Sequential Search

• You are required to find a given K in an array with a scale of n sequentially

• Best situation
  • The first element of the array is K
  • You only need to check one element

• Worst situation
  • K is the last element of the array
  • You need to check all the n elements of the array.
Find value k sequentially—the average case

• If value is distributed with equal probability
  - The probability that K occurs in every position is 1/n

• The average cost is O(n)

\[
\frac{1 + 2 + \ldots + n}{n} = \frac{n + 1}{2}
\]
1.4 Complexity analysis of algorithm

Find value k sequentially—-the average case

- Distributed with different probability
  - Probability that K occurs in position 1 is 1/2
  - Probability that K occurs in position 2 is 1/4
  - Probability that K occurs in other positions are all
    \[
    \frac{1 - \frac{1}{2} - \frac{1}{4}}{n - 2} = \frac{1}{4(n - 2)}
    \]

- The average cost is O(n)
  \[
  \frac{1}{2} + \frac{2}{4} + \frac{3 + \ldots + n}{4(n - 2)} = 1 + \frac{n(n + 1) - 6}{8(n - 2)} = 1 + \frac{n + 3}{8}
  \]
1.3 Algorithm

Binary search

For sequential linear list that is in order

• $K_{mid}$: The value of the element that is in the middle of the array
  - If $k_{mid} = k$, the search is successful
  - If $k_{mid} > k$, the search continues in the left half
  - Otherwise, if $k_{mid} < k$, You can ignore the part that before mid and search will go on in the right part

• Fast
  - $k_{mid} = k$, search will be ended up
  - $K_{mid} \neq k$, reduce half of the searching range at least
1.4 Complexity analysis of algorithm

Performance analysis of binary search

- The largest search length \( \lceil \log_2 (n + 1) \rceil \)
- The search length of the situation that failed is \( \lfloor \log_2 (n + 1) \rfloor \) or \( \lceil \log_2 (n + 1) \rceil \)
- The average cost is \( \mathcal{O}(\log n) \)
- In complexity analysis of algorithm
  - The base of \( \log n \) is 2
  - When the base changed, the magnitude of algorithm will not change
Time/Space tradeoff

• Data structure
  - A certain space to store every data item
  - A certain amount of time to perform a single basic operation

• The cost and benefit
  - limit of time and space
  - Software engineering
1.4 Complexity analysis of algorithm

The space-time tradeoffs

- Increasing the space overhead may improve the algorithm's time overhead
- To save space, often need to increase the operation time
Selecting data structure and algorithm

• You need to analyze the problem carefully
  – Especially the logic relations and data types involved in the process of solving problems—problem abstraction, data abstraction
  – Preliminary design of data structure often precede the algorithm design

• Note the data structure of scalability
  – Consider when the size of input data changes, whether data structure is able to adapt to the evolution and expansion of problem solving
Question: Selecting data structure and algorithm

- Goal of problem solving?
- Process of choosing data structure and algorithm?
Question: three elements of data structure

Which of the structures below are logical structure and has nothing to do with the storage and operation().

A. Sequential table  B. Hash table  
C. Linear list  D. Single linked list

The following terms (_____ ) has nothing to do with the storage of data.

A. Sequential table  B. Linked list  
C. Queue  D. Circular linked list
Data Structures and Algorithms

Thanks

the National Elaborate Course (Only available for IPs in China)
http://www.jpk.pku.edu.cn/pkujpk/course/sjjg/

Ming Zhang, Tengjiao Wang and Haiyan Zhao
Higher Education Press, 2008.6 (awarded as the "Eleventh Five-Year" national planning textbook)