Neuronal Dynamics: Computational Neuroscience of Single Neurons

Week 5 – Variability and Noise:
The question of the neural code

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Week 5 – part 4b: Membrane potential fluctuations

- 5.1 Variability of spike trains - experiments
- 5.2 Sources of Variability? - Is variability equal to noise?
- 5.3 Three definitions of Rate code - Poisson Model
- 5.4 Stochastic spike arrival - Membrane potential fluctuations
- 5.5. Stochastic spike firing - subthreshold and superthreshold
Week 5 – part 4b: Membrane potential fluctuations

- 5.1 Variability of spike trains
  - experiments
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  - Is variability equal to noise?
- 5.3 Three definitions of Rate code
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  - Membrane potential fluctuations
- 5.5 Stochastic spike firing
  - subthreshold and superthreshold
Spontaneous activity \textit{in vivo}

Variability of membrane potential?

awake mouse, freely whisking,

\textit{Crochet et al.}, 2011
Neuronal Dynamics – 5.4b. Fluctuations of potential

**Synaptic current pulses of shape \( \alpha \)**

\[ R I^\text{syn} (t) = \sum_k w_k \sum_f \alpha(t - t_k^f) \]

**Passive membrane**

\[ \tau \frac{d}{dt} u = -(u - u_{\text{rest}}) + R I^\text{syn} (t) \]

→ Fluctuating potential

**Fluctuating potential**

\[ I^\text{syn} (t) = I_0 + I^{\text{fluct}} (t) \]

**Fluctuating input current**
Neuronal Dynamics – 5.4b. Fluctuations of potential

\[ \langle \Delta u(t) \Delta u(t) \rangle = \langle u(t)u(t) \rangle - \langle u(t) \rangle^2 = \]

Input: step + fluctuations
Neuronal Dynamics – 5.4b. Calculating autocorrelations

Autocorrelation

\[ \langle x(t)x(t') \rangle = \sum_f \int dt' f(t-t') \delta(t'-t_k') \]

\[ = \int dt' f(t-t') S(t') \]

Mean:

\[ \langle x(t) \rangle = \int dt' f(t-t') \langle S(t') \rangle \]

\[ \langle x(t) \rangle = \int ds f(s) \rho_0 \]

rate of homogeneous Poisson process

\[ \langle x(t)x(t') \rangle = \int dt' \int dt'' f(t-t') f(t'-t'') \langle S(t') S(t'') \rangle \]
Probability of spike in time step:

\[ P_F = \rho_0 \Delta t \]

Autocorrelation (continuous time)

\[ \langle S(t)S(t') \rangle = \rho_0 \delta(t - t') + [\rho_0]^2 \]
for a passive membrane, we can analytically predict the amplitude of membrane potential fluctuations

Leaky integrate-and-fire in subthreshold regime
**Stochastic spike arrival:**
for a passive membrane, we can analytically predict the amplitude of membrane potential fluctuations

*Leaky integrate-and-fire in subthreshold regime*

Passive membrane fluctuating potential

\[ u(t) = \sum_k w_k \sum_f \epsilon(t' - t'_f) \]
\[ = \sum_k w_k \int dt' \epsilon(t - t') S_k(t') \]

\[ \langle \Delta u(t) \Delta u(t) \rangle = \langle [u(t)]^2 \rangle - \langle u(t) \rangle^2 \]
Neuronal Dynamics – 5.4b. Fluctuation of potential

Passive membrane
\[ u(t) = \sum_k w_k \sum_f \varepsilon(t' - t_k^f) \]
\[ = \sum_k w_k \int dt' \varepsilon(t - t') S_k(t') \]

Fluctuations of potential
\[ \langle [\Delta u(t)]^2 \rangle = \langle [u(t)]^2 \rangle - \langle u(t) \rangle^2 \]
A linear (=passive) membrane has a potential given by

\[
u(t) = \sum_f \int dt' f(t - t') \delta(t' - t_k^f) + a
\]

Suppose the neuronal dynamics are given by

\[
\tau \frac{d}{dt} u = -(u - u_{rest}) + q \sum_f \delta(t - t_k^f)
\]

[ ] the filter \( f \) is exponential with time constant \( \tau \)
[ ] the constant \( a \) is equal to the time constant \( \tau \)
[ ] the constant \( a \) is equal to \( u_{rest} \)
[ ] the amplitude of the filter \( f \) is \( q \)
[ ] the amplitude of the filter \( f \) is \( u_{rest} \)