DSAx Part 5
Debt Sustainability Analysis under Uncertainty

Part 5 Unit 1:
Learning Objectives and Structure of Part 5
Emphasize that debt sustainability boils down to a judgment under uncertainty.

Judgment is essentially about the capacity to generate and sustain primary surpluses (“fiscal behavior”).

Introduce a method incorporating both uncertainty and fiscal behavior.

Discuss the policy implications, notably for the definition of safe debt limits.

Structure of Part 5

✓ Unit 1: Overview
✓ Unit 2: The importance of fiscal behavior
✓ Unit 3: Fiscal behavior and sustainability: a model
✓ Unit 4: Fiscal behavior and sustainability under uncertainty
✓ Unit 5: Stochastic simulations: introduction
### Structure of Part 5

- **Unit 6: Introducing VARs**
- **Unit 7: Framework for stochastic DSA**
- **Unit 8: Fan charts, probabilistic vulnerability indicators**
- **Unit 9: Customizing stochastic analysis: an example**
- **Unit 10: Wrapping up**

### Part 5 Unit 2:

**The Importance of Fiscal Behavior**
UNIT OBJECTIVES

- Remind what we think we know about sustainability.
- Raise our awareness of the operational challenges.
- Introduce the notion of fiscal reaction function to model fiscal behavior.

UNIT OUTLINE

- Debt sustainability: does it have a meaning?
- Challenges related to the forward-looking nature of sustainability and to uncertainty
- Introduction to the notion of fiscal reaction function.
Part 5 Unit 2: Lecture 1

Recall What We Mean by Debt Sustainability

What Do We Know about Debt Sustainability?

- A simple concept (solvency)...
- ... But an operational challenge
- Fiscal behavior—in particular the primary balance—is essential
- There is significant uncertainty surrounding assessments
Debt Sustainability: Quotes

“Debt sustainability is one of the most used and abused concepts in recent discussions on preventing and resolving sovereign debt crises. [...] It] is an art rather than a science, and involves a large number of alternative methodologies.”

Sturzenegger and Zettelmeyer, 2006

Debt Sustainability: Quotes

“[...] fiscal sustainability has been an often-voiced concern in the IMF. But just as elsewhere, the precise concern is sometimes unclear because the term fiscal sustainability does not have an exact meaning.”

Chalk and Hemming, 2000
What is Debt Sustainability?

- **Broad Consensus**: public debt is sustainable if the public sector is **solvent**.

  - **Solvency** implies that debt cannot exceed the net present value of future primary surpluses!

- Intertemporal condition linking present public debt to future budget balances → it is entirely forward-looking.

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RECAP

- Sustainability is anchored in solvency
- Solvency constrains what future primary balances can be
Part 5 Unit 2: Lecture 2

Debt Sustainability: Operational Challenges

OUTLINE

- Good and bad ways to be solvent
- Sustainability = exclude the bad ways
- Sustainability is a judgment
Good and Bad Ways to Be Solvent

✓ **Ex-post**, the intertemporal budget constraint is always fulfilled
  ➡️ Sovereign debtor can align present liabilities and future primary surplus at any time.

✓ **Good ways and bad ways to be solvent:**
  ➡️ Bad ➔ reducing current liabilities through default/unexpected inflation, restructuring;
  ➡️ Good ➔ commitment to higher future primary balances.

Sustainability vs. Solvency

✓ **Ex-ante**, public debt is deemed sustainable if one believes that the sovereign debtor will only use good ways to preserve solvency.

✓ **Hence sustainability is (much) more demanding than solvency** because it precludes particular policies/actions (default, restructuring, inflation tax).
Implications

- Sustainability “has no precise meaning.”
- The link between sustainability and solvency is loose.

Any debt sustainability assessment is a judgment about government capacity to rely on “good ways” to remain solvent.

Need tools to assess plausible fiscal behavior in the future.
Uncertainty

This judgment is probabilistic in nature because it concerns the stream of all future primary balances!

Need tools to measure the extent of uncertainty (how likely are we to be wrong?)

Next Steps

Go back to pragmatic notion of sustainability:

- Non-explosive path for debt ratio.

Linking fiscal behavior to sustainability: the “reaction function.”

Government concerns for sustainability and the reaction function.
RECAP

- Solvency is too weak a condition to serve as a concrete guide for policy.
- Sustainability is a more restrictive condition that only allows for good ways to remain solvent.
- Assessing ex-ante the capacity to stick to these good ways is a probabilistic judgment.

Part 5 Unit 2: Lecture 3

What is a Reaction Function?
Definition of Reaction Function

A “reaction function” captures the systematic response of the primary balance to certain variables

- Not a normative approach: no optimization
- Positive approach: empirical description of average patterns in fiscal policy choices
Which Variables to Include

- **Primary balance:**
  - It is closer to actual behavior than overall balance
- **Systematic response to**
  - Output gap,... → counter- vs. pro-cyclical
  - Debt level → concern for debt dynamics
- **Systematic/average response of the primary balance to past debt is critical for debt dynamics**

The Reaction Function Model

- **Reaction function:**
  \[ p_t = F(d_{t-1}, X_t) + \varepsilon_t \]

  Primary balance in \( t \) responds to existing debt and other variables

  This is essential for debt dynamics
Reaction Function and Sustainability

Recall that: \[ d_{t+1} = \left[ \frac{1 + r_{t+1}}{1 + g_{t+1}} \right] d_t - p_{t+1} \]

Hence, the debt ratio to GDP moves as follows:

\[ \Delta d_{t+1} = \left( \frac{r_{t+1} - g_{t+1}}{1 + g_{t+1}} \right) d_t - F(d_t, X_{t+1}) \]

Snowball effect  Fiscal behavior

Reaction Function and sustainability

It is clear that the evolution of the debt ratio depends on whether attention to sustainability dominates the snowball effect.

Recall: snowball effect is the speed at which the debt ratio would automatically grow (or fall...) if the primary balance was zero.
Concern for sustainability is a "marginal" concept. What matters is not the absolute level of the primary balance but its sensitivity to a change in public debt. Bohn (1998) shows that a positive response of the primary balance to debt is sufficient for solvency.

What about sustainability? Stronger condition → See this in a diagram.

Now, we know the workhorse model to capture fiscal behavior. Reaction function. We understand the link between debt sustainability and the reaction function.
We revisited the link between solvency and sustainability.

We showed that sustainability rests on an intrinsically probabilistic judgment about fiscal behavior.

We saw how to capture fiscal behavior in a formal model.

We showed that this model is related to debt dynamics.

Part 5 Unit 3

Fiscal Behavior and Sustainability: a Model
UNIT OBJECTIVES

- Formal framework linking sustainability to the reaction function
- What causes explosive debt dynamics?

UNIT OUTLINE

- A diagrammatic framework
- Stable vs. unstable equilibrium
Part 5 Unit 3: Lecture 1

Diagrammatic framework

- Diagram linking primary balance to debt.
- Representing the snowball effect
- Representing the reaction function
The diagram collects combinations of positive debt levels and primary surpluses.

Next step: identify combinations consistent with stable or decreasing debt levels and those leading to ever increasing debt levels.

\[ P \]

\[ d \]

Simple diagram

\[ X(d, p) \]

A demarcation line

Assume: positive snowball effect \((r > g)\)

Demarcation line along which primary surplus is keeping debt ratio stable over time.

\[ \Delta d = 0 \Leftrightarrow p = \left( \frac{r-g}{1+g} \right) d \]

Slope = \((r - g)/(1 + g)\)
Above the demarcation line, primary surplus exceeds the debt-stabilizing level: debt is falling over time.

\[ \Delta d = 0 \]

Slope = \frac{(r - g)}{(1+g)}

Below the demarcation line, primary surplus falls short of the debt-stabilizing level: debt is rising over time.

\[ \Delta d = 0 \]

Slope = \frac{(r - g)}{(1+g)}
Lower interest rate

Lower growth-adjusted interest rate reduces the zone (likelihood) of rising debt.

Higher interest rate

Higher growth-adjusted interest rate expands the zone (likelihood) of rising debt.
**The reaction function**

**AA schedule:** average fiscal behavior:  
\[ p_t = \alpha + \beta X_t + \gamma d_{t-1} + \epsilon_t \]

Slope of AA = strength of the primary balance response to a change in debt

\[ \text{Diagram representing combinations of } p \text{ and } d \]

**RECAP**

- Demarcation line defining zones with rising and falling debt.
- Higher growth-adjusted interest rate \((r-g)\) makes rising debt more likely.
- Locate the reaction function.
Part 5 Unit 3: Lecture 2

The equilibrium: stable or unstable?

- Combine fiscal behavior and the demarcation line
- Stable vs. unstable equilibrium, and debt sustainability
- Lessons
**Stable equilibrium**

Stable equilibrium = reversion to initial debt level after a shock.
Condition: response of the primary balance to the public debt must be strong enough:

\[ \gamma \geq (r - g) \]

Fiscal policy response to debt is strong enough to guarantee convergence to a certain debt level.

**Unstable equilibrium**

Unstable equilibrium = shock leads to ever increasing or decreasing debt.
Condition: response of the primary balance to the public debt is too weak:

\[ \gamma < (r - g) \]
Unstable equilibrium can be used to determine a debt limit. Debt limit = level beyond which government loses control of debt dynamics → explosive path.

Now this debt level is the limit of a danger zone.

Lessons

- A strong response of the primary balance to debt implies mean-reversion of debt → debt is always sustainable.

- A weak response implies a debt limit beyond which explosive paths occur → beyond a certain threshold, debt is unsustainable.
Lessons

What makes the response of $p$ to $d$ “strong”? $\gamma \geq (r - g)$

Concretely?

- Assume $r$ is 3 percent and growth is 2 percent.
- Then on average, an increase in $d$ by 10 percentage point of GDP requires a minimum increase in $p$ by 0.1 percent of GDP.
- Condition is much harder to satisfy if $r$ rises or $g$ falls.

RECAP

- A sufficiently strong response of the primary balance is essential to guarantee sustainability.
- A strong response = average increment of the primary balance to an increase in debt is greater than $r - g$.
- A weak response—although technically sufficient for solvency—cannot rule out explosive debt paths.
Part 5 Unit 4

Fiscal Behavior and Sustainability under Uncertainty

UNIT OUTLINE

- Various sources of uncertainty (r-g but also behavior).
- Specific cases → multiple equilibria: fatigue and market response
Part 5 Unit 4: Lecture 1

Sources of uncertainty

OUTLINE

Use the diagrammatic model to show the impact of uncertainty.

Illustrate:
- Uncertainty about r-g
- Uncertainty about fiscal behavior
Uncertainty about growth and interest rates

Introducing uncertainty (r-g)

Uncertainty about r and g moves the demarcation line.

The strong fiscal policy response now ensures that Public debt will tend to revert to an interval of values.

Introducing uncertainty (p)

Uncertainty about the reaction function.

Here, uncertainty is not large enough to lead to unstable equilibria.
Introducing uncertainty (p)

Here, uncertainty is large enough to make an unstable equilibrium possible even at low debt levels.

We can have a low debt equilibrium that is unstable and a higher debt equilibrium that is stable. Assessing capacity to generate surpluses is essential for DSA!

Uncertainty about r-g broadens the range of debt levels to which one can converge.

Uncertainty about fiscal behavior can dramatically affect assessments:

- Low-debt unstable equilibrium can be as plausible as a high-debt stable equilibrium.
Part 5 Unit 4: Lecture 2

Multiple equilibria

- Notion of multiple equilibria
- Two plausible cases in which this can occur
Multiple equilibria

Basic concept: for a given fiscal behavior, and a given \( r-g \), a stable and an unstable equilibrium coexist.

Exogenous shocks to \( r \) and \( g \) can cause shifts from a stable to an unstable equilibrium.

Case #1: adjustment fatigue → upper limit to feasible primary balance.

Case #2: risk premium increases along with debt level.

Adjustment fatigue

Beyond that point, fiscal policy stops responding to debt: danger of explosive paths

safe threshold?
Interpretation

✓ For given r-g: stable (low-debt) equilibrium and an unstable (high-debt) equilibrium → Upper limit on p implies a debt threshold beyond which sustainability is a concern.

✓ Forming a view on the maximum primary balance a government can sustain helps pin down a “safe” debt threshold (below which debt is always sustainable, even under bad realizations of r-g).

Risk premia: r rises with d

Volatility implies serious uncertainty on the debt level beyond which things get out of control

Stable equilibria (low debt)

Unstable equilibria

Safe threshold?
Interpretation

- Again: low debt levels remain sustainable under high realizations of \( r-g \).
- Safe threshold? Somewhere below the level corresponding to the unstable equilibrium under a high realization of \( r-g \).

RECAP

- Multiple equilibria can arise from adjustment fatigue and risk premia.
- Shocks can cause shifts from stable to unstable equilibrium.
- Threshold for safe debt levels? Low enough to ensure that even in bad situations (e.g., high \( r-g \)), debt converges to a stable equilibrium.
Uncertainty plays a critical role in DSA. It makes high debt levels more likely to be unsustainable.

Assessment about debt sustainability must be more reliable than winning a toss!

Being explicit about probabilities has a value.

Part 5 Unit 5
Stochastic simulations: Introduction
UNIT OBJECTIVES

- Introduce the principles of stochastic simulations
- The value added of stochastic methods.

UNIT OUTLINE

- A simple example of stochastic simulations
- Stress tests vs. stochastic simulations
Part 5 Unit 5: Lecture 1

Stochastic simulations: a simple example

OUTLINE

- Definition
- Simple example
Definition

A simulation is said to be stochastic when it involves a variable or a set of variables that are random.

Random variables can change with a certain probability.

Running a large number of stochastic simulations help map the uncertainty surrounding any variable of interest.

A simple example

Assume that the “true” relation linking the rate of growth of tax revenues and the rate of growth of GDP is:

\[ G(t) = \alpha + 1 \times G(y). \]

In reality, this true model is unknowable → it has to be estimated.

How do we model the uncertainty of the real world?

Simulate \( G(t) = G(y) + \epsilon \), where \( \epsilon \) is random.
A simple example

Randomized $G(t)$ series against the “true” $G(t)$. Assume $N(0,2)$.

An OLS regression of $G(t)$ on $G(y)$ gives a coefficient of 0.88, not 1!

A different simulation will give a different result (random draw of $\varepsilon$ will be different)
Run a large number of such simulations: distributions around each data gives an idea of the degree of uncertainty.

A stochastic simulation "randomizes" a given data series.

A large number of simulations give a distribution around each data point.

Display bands characterizing uncertainty.
Part 5 Unit 5: Lecture 2

Stress tests vs. stochastic simulations

OUTLINE

- Limits of stress tests
- Pros and cons of stochastic simulations.
Stress tests

- Focus is on individual simulations of unfavorable shocks or combinations of shocks.
- Issue: define a sufficiently stressful scenario, while remaining plausible.

Limits

- Not based on the solvency condition. Simply look at the consequences of continuing certain policies on debt dynamics.
- Each individual test has a zero probability of occurrence
  - cannot distinguish plausible developments from genuine tail risks.
  - one might pay undue attention to seemingly plausible scenarios that are in fact quite unlikely → risk to get it wrong
    - Markets: speculative attacks
    - Policymakers: complacency or panic.
Limits

Stress tests generally ignore:

- underlying correlations among variables.
- Endogenous response of policymakers to shocks but also to debt itself.

Consequence:

- Unproductive debate on the design of stress test rather than risk itself.

Stochastic simulations

Pros:

- Aim at giving the full picture about uncertainty surrounding debt projections
- Allow for explicitly probabilistic assessments of explosive debt paths or risk of exceeding critical thresholds

Cons:

- Can be demanding technically and in terms of required data
- "Black box" effect: hard to trace intuitively what is at play.
Stress tests are simple, but may be too simple → chance that discussions they trigger be about the design of the test itself rather than the risk to debt.

Stochastic simulations can offer the full picture, but discussions affected by black box effect.

Next step: to simulate shocks, we need a joint distribution → VAR model.

Part 5 Unit 6
Introducing VARs
What is a VAR?

What do we take from it for our purpose?

Part 5 Unit 6: Lecture 1

What is a VAR?
What is a VAR?

- Linear dynamic econometric model linking a set of economic variables without underlying theory.
- All the variables of interest are explained by their own history (lags) and the history of all other variables.
- Main use: forecasting and tracing responses to policy decisions.

Example VAR

\[ X1_t = a + bX1_{t-1} + c X1_{t-2} + d X2_{t-1} + e X2_{t-2} + u1_t \]
\[ X2_t = f + gX1_{t-1} + h X1_{t-2} + i X2_{t-1} + j X2_{t-2} + u2_t \]

\( u1_t \) and \( u2_t \) are the reduced form disturbances or errors \( \rightarrow \) they are generally correlated.
Tips on Estimating VARs

✓ VARs can be estimated by regressing each equation in the system individually.
✓ Lag length matters.
✓ Variables in the VAR should either be stationary or cointegrated to ensure unbiased coefficients.
✓ VAR models have many coefficients to be estimated → need quite a lot of data (min. 30 to 40 obs).

Part 5 Unit 6: Lecture 2

Useful outputs from the VAR?
What do we get from the VAR?

- Model of how the relevant variables are linked: $r, g, p, ...$
- Forecasting tool for these variables:
  - Captures relationship among them
  - Dynamics
- Estimate of the joint distribution of shocks $\rightarrow$ need for structural innovations.

Obtaining Structural Innovations

- Reduced form disturbance terms (the $u$’s) are in general correlated across equations. Their joint distribution is the variance-covariance matrix $\Omega$.
- As we are interested in “pure shocks,” we must transform the reduced-form disturbances into uncorrelated structural innovations.
Obtaining Structural Innovations

- This requires identifying restrictions.
- A simple method involves “ordering” the variables in the VAR.
- This means restricting the effect of structural innovations on the reduced form disturbance terms of other variables to zero (Sims, 1980).

Ordering in a Two-variable System

U1 = αe1
U2 = βe1 + δe2

α, β, δ, e1 and e2 can be found based on Ω and U1 and U2.
Ordering in VARs

- When using matrices, one refers to the Cholesky decomposition of the variance-covariance matrix $\Omega$.

$$\Omega = SS'$$

- $S$ is a lower-triangular matrix.
- $S'$ is its transpose.

- The structural innovations can be recovered from the reduced form disturbances using $u = Se$, where $u$ and $e$ are vectors.

UNIT RECAP

VARs are useful for forecasting.
VARs give an idea of shocks affecting variables.
Combining these two features is key for a credible stochastic simulation framework.
Present the building blocks of a full fledged stochastic simulation framework for DSA

Framework has 3 objectives:

- Incorporate fiscal behavior
- Applicability to a broad range of countries
- Not too demanding in terms of fiscal data.
Part 5 Unit 7: Lecture 1

Characterizing fiscal behavior
(the fiscal block)
Estimating Primary Surplus
Reaction Functions

- Basic issue: meaningful fiscal data has an annual frequency → yearly data.
- Initial question: country-specific estimates or not?
  - Country-specific estimates:
    - Require reasonably long time series, which may not exist,
    - ...and stable patterns of behavior over time.
    - Assumption that past behavior is bound to repeat itself in the future.
  - Panel (group of countries):
    - More observations (degrees of freedom)
    - Allow to base fiscal behavior assessment on average patterns among a given group of countries (less dependence on a country’s own history).
Specification

- Basic specification (panel):

\[ p_{i,t} = \alpha_0 + \rho d_{i,t-1} + \gamma \text{gap}_{i,t} + X_i \beta + \eta_i + \epsilon_{i,t}, \quad t = 1, \ldots, T, i = 1, \ldots, N \]

- Alternative: allow for the response to debt accumulation to weaken once debt exceeds a certain level and for asymmetric response to business cycle.

Estimation

- Panel: combine cross-country and time-series dimension

  - More observations
  - Estimate average reaction function among a group of countries (fixed-effect model).
  - Generalizes Bohn solvency test.

- Two panels used here:
  - G20 advanced economies (except Japan), Belgium, Greece, Ireland, and Spain.
Key results

- On average, fiscal behavior is consistent with solvency:
  - the slope $\gamma = 0.03$ to $0.06$.
  - in the panel of emerging markets, response weakens after debt exceeds 50 percent of GDP, but remains positive.
- Primary balance is broadly acyclical in emerging markets and countercyclical in bad times in advanced economies.
- A negative shock on growth generally affects the primary balance.

RECAP

- Opt for annual fiscal data and panel approach.
- Fiscal behavior:
  - is generally consistent with solvency.
  - responds to economic shocks.
- All this matters for assessing risks to debt sustainability.
Part 5 Unit 7: Lecture 2

The distribution of shocks (the economic block)

OUTLINE

- Using the VAR
- Merging VAR, fiscal behavior and debt projection
- Advantages and limitations
Building blocks

Unrestricted VAR model:

\[ Y_t = \gamma_0 + \sum_{k=1}^{\nu} \gamma_k Y_{t-k} + \xi_t \]

with \( \xi_t \), a vector of reduced-form residuals:

\[ \xi_t \sim N(0,\Omega) \]

Estimation: quarterly data (to have enough observations).

Ordering of the VAR is \( Y_t = (r,\pi^m, r, g, z, \ldots) \) to obtain structural shocks.

From the VAR, we get:

- 1000 random draws of 5-year sequences (20 quarters) of structural shocks. Based on joint-normal distribution.
- Use VAR for 1000 projections for \( Y \) consistent with each sequence of shocks.

Issues:

- We have quarterly shocks and projections. Need to annualize all the projections that enter in the debt dynamics equation.
- The reaction function (fiscal policy) not in the VAR. Separate panel estimation.

Implied restriction: fiscal policy itself has no impact on projections for \( r, g, \ldots \).
Building blocks

- Debt identity equation combine annualized VAR projections and the fiscal reaction function.
  
  \[ d_i = (1 + g_i)^{-1}\left[\left(1 + r_i^w\right)\left(1 + \Delta z_i\right)d_i^* + (1 + r_i)\tilde{d}_{i-1}\right] - p_i \]

- For each projection of a debt path, we need the predicted value for \( p \) (reaction function).

- Example: \( \hat{p}_{i,t+\tau} = \Lambda_{i,t+\tau} + \hat{\rho}d_{i,t+\tau-1} + \hat{\gamma} \text{ygap}_{i,t+\tau} + \phi_{i,t+\tau} \)

- Hats denote predicted values or estimated coefficients (from panel estimation).

- The last term \( \Phi \) is a shock on fiscal policy itself. It is independent of other shocks.

Calibrating the reaction function

- The term \( \Lambda \) captures the “base” value of \( p \).
  
  \[ \Lambda_{i,t+\tau} = \hat{p}_{i,t} - \hat{\rho}d_{i,t-1} - \hat{\gamma} \text{ygap}_{i,t} + \kappa_{i,t+\tau} \]

- The base value is the value predicted by the model in the last year before projection \( (t) \) filtered of the effect of the debt level and the output gap for that year.

- Question: Is the model forecast error for year \( t \) likely to be temporary or permanent?
  
  - If permanent \( \Rightarrow \) “constant underlying policy” scenario (CUP) \( \kappa_{i,t+\tau} = 0 \)
  - If temporary \( \Rightarrow \) “predicted policy” scenario \( \kappa_{i,t+\tau} = -\hat{\epsilon}_{i,t} \)
Presented the two blocks of the simulation model (Celasun, Debrun and Ostry, 2007)

Estimated reaction function: using annual data and panel approach.

Combined VAR-based stochastic simulation tools with reaction function and debt identity.

Part 5 Unit 8

Fan charts and probabilistic sustainability indicators
How to present stochastic simulation results?
Build meaningful debt sustainability indicators.

Introducing fan charts.
Other probabilistic indicators.
Part 5 Unit 8: Lecture 1

Fan charts

**OBJECTIVES**

- Organize output from stochastic simulations.
- Interpret fan chart.
Results from simulations

After simulations, we have a large number of debt paths covering a range of shocks and policy responses based on consistent projections of all relevant variables.

For each year, we have 1000 debt levels → 5000 figures if we have projections for 5 years.

The output looks like this! (Each column is a year, and there 1000 lines!)

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Mapping risk

For each year, we can build a histogram: frequency distribution for one year only.

Distribution skewed towards high debt ratios despite normal shocks! Why? Snowball effect grows with debt level.

Mapping uncertainty

How to fully capture uncertainty around the baseline debt path?
From histogram to fan chart

“Collapse histogram” into frequency bands of different colors → fan chart.

Fan chart vs. stress test

Real example: South Africa in 2005. Stress test appears to underestimate the risk.
Acid test

Can fan chart really alert of impending crisis?
Run the framework in the past just before two major crisis episodes → Your conclusions?

Turkey at end-1999

Argentina at end-2000

Stochastic simulations give large output.
Organize data in a meaningful way to map uncertainty around baseline projection:
   Histograms.
   Fan charts.
Part 5 Unit 8: Lecture 2

Other probabilistic indicators

- Going beyond fan charts → explicit probabilities.
- Review specific indicators (many others could be built).
Beyond fan charts

- Fan charts give the broad picture.
- Stochastic simulations provide information to build more specific indicators.
- Three examples:
  - Likelihood of a good event: public debt ratio will stabilize by year 2017.
  - Assessing upside risk: what is the largest increase in debt that has a more than 10 percent chance of materializing by a certain date?
  - Numerical indicator that increases with likelihood of good event and probability that bad event does not occur.

Example 1

- Probability that debt stabilizes (i.e. stops rising or decline by 2017). Below: first year of projection is 2013 (based on October 2012 WEO vintage).
Aside

Recent crisis raises question of a new normal in r and g (both lower). Should we rely on r-g “steady state” embedded in the VAR?

Example 2

What is the maximum increase in debt that has 10 percent chance or more of materializing by 2016? (Same starting date as in example 1). quantifies how bad an adverse scenario could really be.
Example 3: synthetic indicator

- Combine probability of good event with non-occurrence of a very bad event.

- Specifically:
  - Probability that debt declines: \( \Pr(d_{t+1} < d_{t+1-1}) \)
  - Probability that debt does not exceed a certain threshold: \( 1 - \Pr(d_{t+1} > (d_t + x)) \)
  - A combination of the above: \( \Pr(d_{t+1} < d_t) \times \left[ 1 - \Pr(d_{t+1} > (d_t + x)) \right] \)
  - Critical value? Up to tolerance for bad event and desirability of good event.
    - Pick 0.4: probability of falling debt of at least 50 percent and a probability of debt rising by 10 percent of GDP over forecast horizon of 20 percent.
    - Critical value depends on: risk aversion, desirability of declining debt/tolerance for some debt increase.

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UNIT RECAP

- Stochastic simulations generate data on the full extent of risks to debt dynamics.
- Fan charts are a visual way to map risk to debt dynamics.
- Output of stochastic simulations allows calculating explicitly probabilistic debt sustainability indicators.
Part 5 Unit 9

Case study: a simple application of stochastic methods

UNIT OBJECTIVES

✓ Compare use of stochastic simulations and stress tests to address a concrete problem.
✓ Show that stochastic simulations can be used in a very simple way to illustrate specific risks.
Part 5 Unit 9: Lecture 1

The case and stress tests

The case

Low income countries:

- Debt relief → solvency is not an immediate concern
- How do we keep our eyes on the ball?

Specifically:

- How should we manage the desirable debt build up over time? → trade off between prudence and ambition.
- How do we deal the higher volatility of aid flows? → trade off between saving windfalls and need to spend (to provide relief to the poor...but also because donors want it!)

Illustration: Ethiopia (Heller and others, 2006).
The case

Poverty reduction and growth require a lot of financing, commitments do not always follow → how should one view medium term planning?

Too ambitious? → risk of quickly slipping back into excessive debt.
Too conservative? → Undue constraints on productive expenditure!

Assessing volatility: stress tests

Liquidity: impact of aid shocks on primary balance (Ethiopia) if the shortfall is fully financed with new debt.

Look at 2 shocks:

- Growth in grants = historical average – 1SD for 2 years. Catch up period of 2 years (to avoid permanent effect of shock).
- 3-year episode of low growth and a catch-up phase of 5 years.

Liquidity risk is non-negligible even for the mild aid shock.
Ethiopia: stress tests

Solvency: impact on debt

- One single shock has a lasting effect on debt.
- But one needs a protracted shock to raise concerns about sustainability.

Issues

- Isolated stress tests convey fairly different messages... and they are both arguably implausible.
- Can a more complete image of risk be obtained without additional complexity or data needs?
Part 5 Unit 9: Lecture 2

Stochastic simulations

The stochastic simulations exercise

- Stress tests are subject to endless debates on calibration.
- Simulate sequences of shocks on aid over 10 years (shocks are zero-meaned and have a variance = historical variance of real aid).
  - No VAR, no large data requirements, no modeling: just random number generator in Excel (same as generating a large number of stress tests).
- Build fan charts.
Ethiopia: fan charts

Liquidity: primary deficit
After 3 years, there are non trivial chances (more than 1 in 10) to have large additional borrowing needs (3+) leading to quick debt accumulation.

Ethiopia: fan charts

Solvency: NPV of debt
Notice: stochastic simulations reflect only shocks on aid inflows → very bad debt outcomes are not unlikely (growth could fall and borrowing costs could rise, along with adverse aid shock).
Stochastic simulations are a flexible tool to assess specific risks.

No need for complex modeling nor large data requirements.