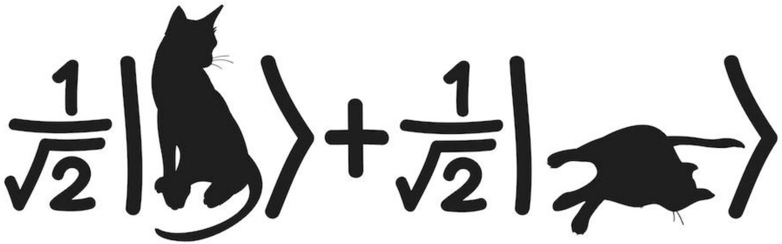
Quantum Mechanics & Quantum Computation



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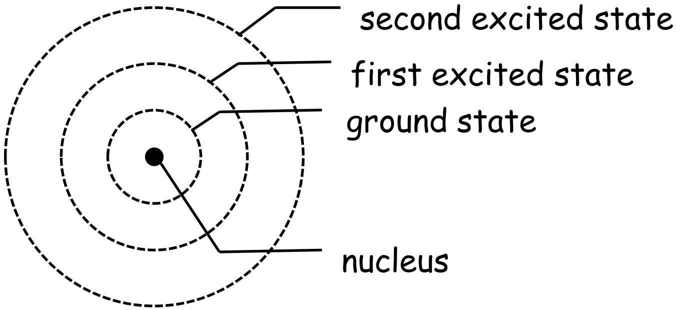
Lecture 2: Qubits & Uncertainty Principle

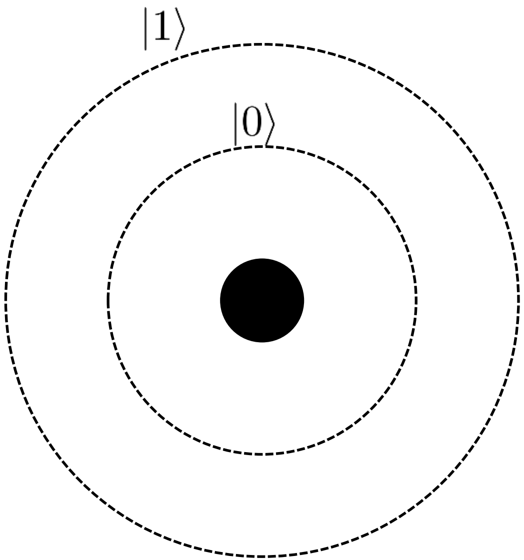
Qubits

Using an electron to represent a bit:

Energy of an electron in an atom



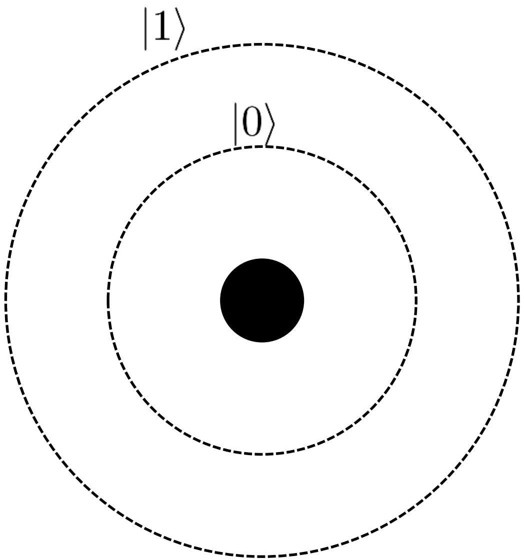
Using an electron to represent a bit:



ground state $\longleftrightarrow$ 0
excited state $\longleftrightarrow$ 1

Ground with prob $1/3$
Excited with prob $2/3$.

Qubit (quantum bit):



complex amplitude:

$$\alpha|0\rangle + \beta|1\rangle$$

ground   excited

example:

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

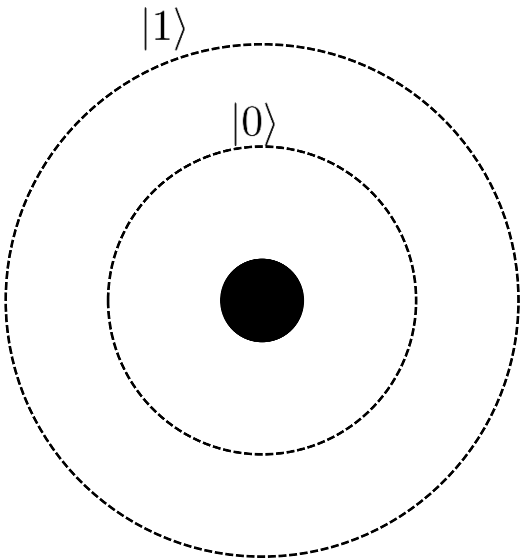
$$|\alpha|^2 + |\beta|^2 = 1$$

$$\underbrace{\left(\frac{1}{2} + \frac{1}{2}i\right)}_{=\alpha}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$\alpha = a + bi \qquad |\alpha| = \sqrt{a^2 + b^2}$

$|\alpha|^2 = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2} \qquad |\beta|^2 = \frac{1}{2}$

Qubit (quantum bit):



$\alpha|0\rangle + \beta|1\rangle$, $|\alpha|^2 + |\beta|^2 = 1$.

Measurement :

ground state $|0\rangle$ wp $|\alpha|^2$

$|1\rangle$ wp $|\beta|^2$

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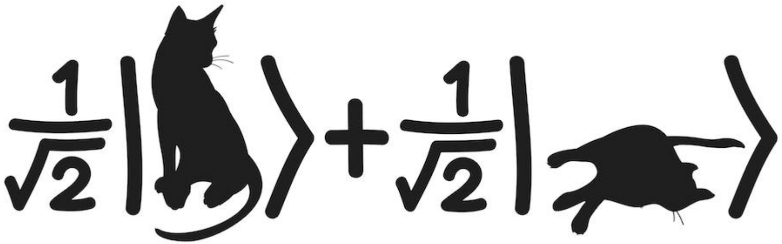

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Lecture 2: Qubits & Uncertainty Principle

Geometric Representation

Geometric Interpretation:

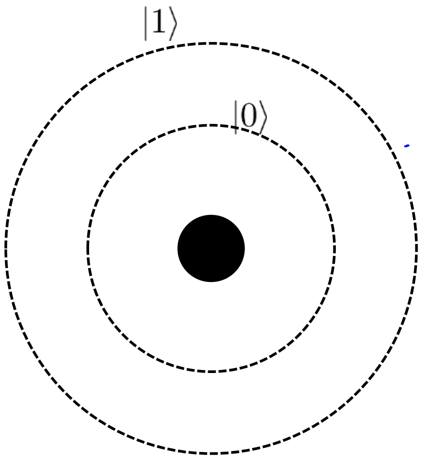
Ket Notation
Dirac

$$\underbrace{\alpha|0\rangle + \beta|1\rangle}_{\begin{pmatrix}\alpha\\ \beta\end{pmatrix}}$$

$\alpha, \beta \in \mathbb{C}$

$|\alpha|^2 + |\beta|^2 = 1.$

$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \longleftrightarrow$ 2 Dimensional complex unit vector.



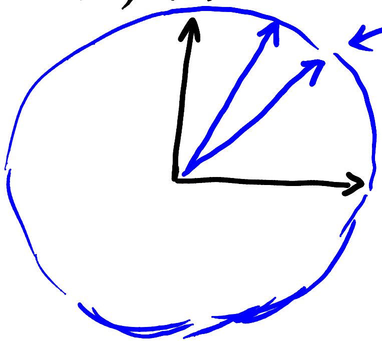
$|0\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}$

$\alpha = 1 \quad \beta = 0$

$|1\rangle = \begin{pmatrix}0\\ 1\end{pmatrix}$

$\alpha = 0 \quad \beta = 1$

$\begin{pmatrix}0\\ 1\end{pmatrix} = |1\rangle$

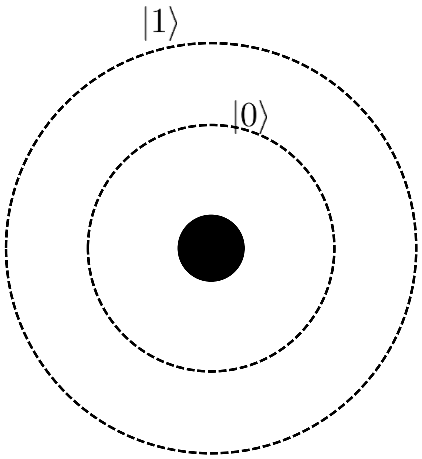


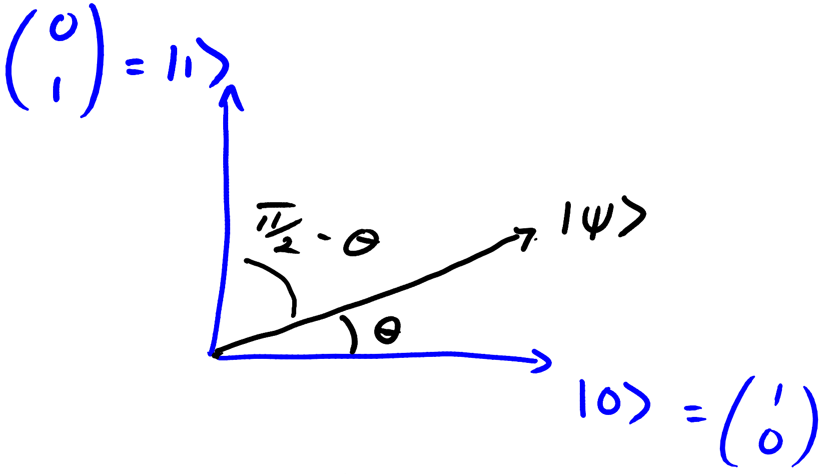
$|0\rangle = \begin{pmatrix}1\\ 0\end{pmatrix}$

$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = \begin{pmatrix}1/\sqrt{2}\\ 1/\sqrt{2}\end{pmatrix}$

$\frac{1}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle$

Measurement:



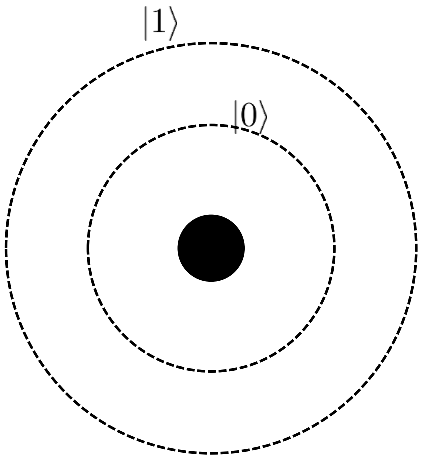


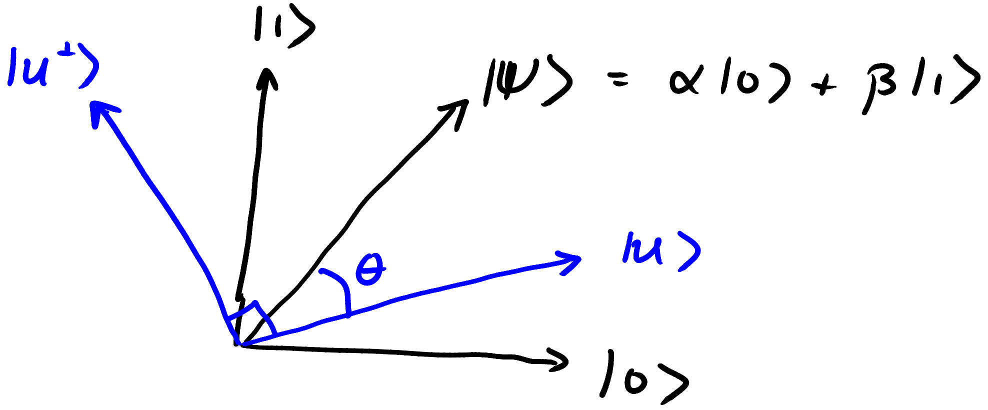
$$|\psi\rangle = \overset{\alpha}{\overset{\shortparallel}{\cos\theta}}\,|0\rangle + \sin\theta\,|1\rangle = \begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}$$

<u>Measure</u> : see $|0\rangle$ wp $\cos^2\theta$

$|1\rangle$ wp $\sin^2\theta = \cos^2\left(\frac{\pi}{2} - \theta\right)$

Measurement in arbitrary basis:





Measure $|\psi\rangle$ in $u, u^\perp$ basis

$$|u\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

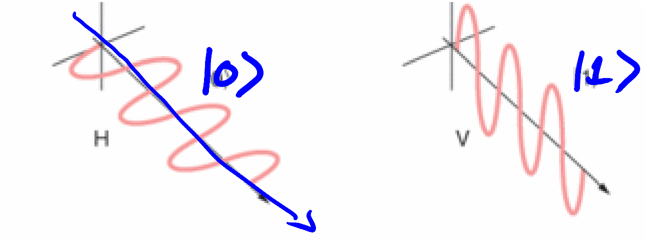
$$|u^\perp\rangle = -\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

see : $|u\rangle$ wp $\cos^2\theta$

$|u^\perp\rangle$ wp $\sin^2\theta$.

Another Example of Qubits

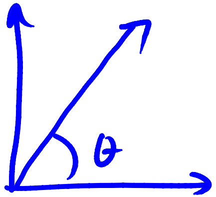
Photon polarization



Horizontally and vertically orientated electric field oscillations.

$$\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$|\uparrow\rangle = |1\rangle$



$|0\rangle$
$\shortparallel$
$|\rightarrow\rangle$

Measuring Polarization

lens

blocked

transmitted

$|\uparrow\rangle$ goes through.

$|\rightarrow\rangle$ gets blocked

$\cos\theta\,|\uparrow\rangle + \sin\theta\,|\rightarrow\rangle$

transmitted wp $\cos^2\theta$

New state $= |\uparrow\rangle$

Blocked wp $\sin^2\theta$.

Measuring Polarization

back $\uparrow$

front $\rightarrow$

$\cos\theta\,|\uparrow\rangle + \sin\theta\,|\rightarrow\rangle$

transmitted wp $\cos^2\theta$

$|\uparrow\rangle$

blocked wp 1.

Middle $\nearrow$

transmitted wp $\frac{1}{2}$

$\frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\rightarrow\rangle$

$1/4$

front $\rightarrow$

transmitted wp $\frac{1}{2}$

$|\rightarrow\rangle$

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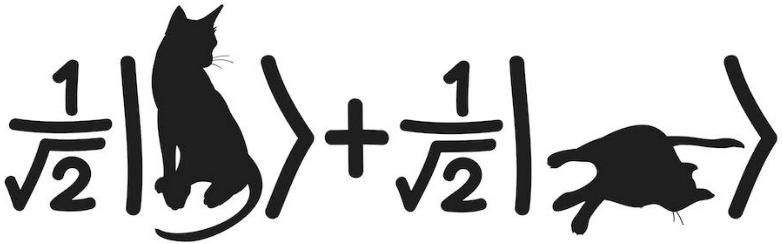

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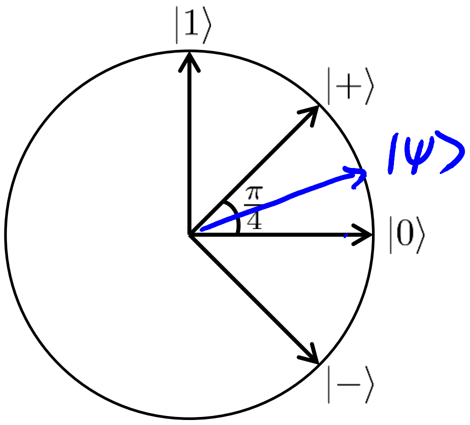
Lecture 2: Qubits & Uncertainty Principle

Uncertainty Principle

Uncertainty principle

One can never know with perfect accuracy both of those two important factors which determine the movement of one of the smallest particles – its position and its velocity.

- Werner Heisenberg



$$|0\rangle, |1\rangle \Big\} \text{ bit}$$

$$\left.\begin{aligned} |+\rangle &= \tfrac{1}{\sqrt{2}}|0\rangle + \tfrac{1}{\sqrt{2}}|1\rangle \\ |-\rangle &= \tfrac{1}{\sqrt{2}}|0\rangle - \tfrac{1}{\sqrt{2}}|1\rangle \end{aligned}\right\} \text{ sign}$$

bit $\longleftrightarrow$ position

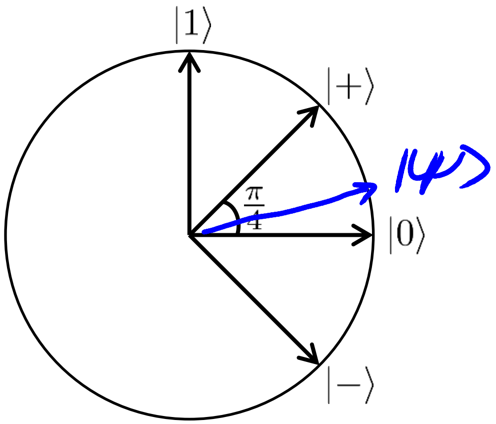
sign $\longleftrightarrow$ velocity/momentum

Uncertainty principle

One can never know with perfect accuracy both of those two important factors which determine the movement of one of the smallest particles – its position and its velocity.

- Werner Heisenberg

- Can we know both bit and sign of a qubit simultaneously?



Bit value known perfectly:

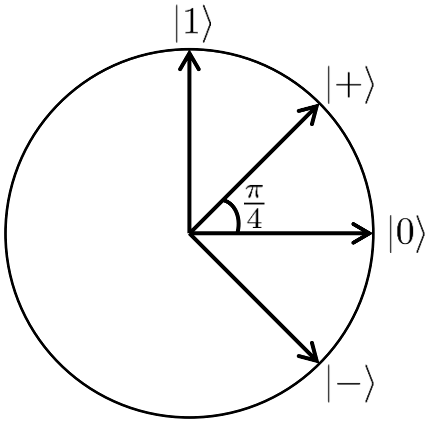
$|0\rangle$ or $|1\rangle$

Sign value perfectly known:

$|+\rangle$ or $|-\rangle$

- To quantify this, can define **spread** of a quantum state.

$$|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle = \beta_0|+\rangle + \beta_1|-\rangle$$



- The spread in standard basis := $S(|\psi\rangle) = |\alpha_0| + |\alpha_1|$.
- The spread in sign basis := $\hat{S}(|\psi\rangle) = |\beta_0| + |\beta_1|$.

$$S(|0\rangle) = 1 + 0 = 1 \qquad \hat{S}(|0\rangle) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$S(|+\rangle) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2} \qquad \hat{S}(|+\rangle) = 1 + 0 = 1$$

- **Uncertainty principle for bit and sign**: $\underline{S(|\psi\rangle)}\,\underline{\hat{S}(|\psi\rangle)} \geq \sqrt{2}$ for any $|\psi\rangle$.

$\vee|$ or $\vee|$

$\sqrt[4]{2}$ $\sqrt[4]{2}$