PROFESSOR PANOS SHIAKOLAS: Let us apply trigonometric principles that we have introduced in the analyses of robotic devices. Our examples or applications will start with the simplest robotic mechanism, which is one-link planar robot. Then, given the link length or the length of this link and the rotation angle in this particular example-- the rotation angle alpha-- we are asked to find the coordinates of the tip of the end of the link in this plane called x and y.

So, how are we going to approach this? I will re-draw the plane, and show the coordinates x and y there. Vectors x and y, that indicated the plane. Then, I will draw the link and the angle of rotation, alpha. The tip of the link, or the tip of the end effector as we call it in robotics is shown here and indicated with the Letter E. We are asked to find the x and y-coordinates.

Well, we could consider this link as the hypotenuse of a right angle triangle, that is formed by drawing perpendicular lines from the tip to the two axes of the coordinate system. So if I draw perpendicular lines, to axis x and axis y, I observe that a right angle triangle is formed. Let us identify this right angle triangle.

Let us call the origin here Letter O. Let us call the tip of the link, Letter E. Let us call the perpendicular to the x-axis, A and the perpendicular to the y-axis, B. So we are forming two triangles, two right angle triangles.

These two triangles are OEB and OEA. We already know the angle of rotation, alpha. And we know the hypotenuse. And we are asked to find the other two sides. Basically OA, A which is the coordinate along the x-axis and OB, which is the coordinate along the y-axis.

If we reflect back to the theory that we introduced, let us write down either the sine or the cosine of an angle. So the sine of angle alpha would be equal to the opposite side, which is AE divided by the hypotenuse which is OE. And for this example, I'm using triangle OEA.

So in this relationship, AE is the desire or unknown coordinate of the end effector in the y direction. And OE is the hypotenuse which is the length. So here, we have this equality, and if we proceed to the next step, we can solve for the y-coordinate, YE is the hypotenuse L times the sine of angle alpha.

How do we find the x-coordinate, or the distance away? Let us take the cosine of angle alpha. The cosine of the angle will be the near perpendicular, which is OA, divided by the hypotenuse, OE. Or OA is the desire or quantity that we are interested in finding. The coordinate along the x-axis and OE is the hypotenuse, which is given by L.

And the next step, if we solve for the unknown quantity, XE, we find this to be L times cosine alpha. So here, in this example, we have use sinusoid and cosine identities to solve for the unknown coordinates XE and LE.

So as this robot is rotating-- as this robot is rotating about the origin O, with an angle alpha, we can find out where it will be. Let us assume that angle alpha is equal to 0 degrees, which means that our robot will be along the x-axis.

So, if we apply this angle alpha into the values of XE and YE, into the expressions of XE and YE, we find that XE is equal to L times cosine of 0 degrees, which is L times cosine of 0. But we know that cosine of 0 degrees is equal to 1, so it's going to be L. And YE is equal to L times sine of 0 degrees. Sine of 0 degrees is 0. So this gives us a y-coordinate of 0. And if we draw these, it does make sense.

We are having a distance along x. If alpha, now, is equal to 90 degrees, where is this mechanism going to be? XE is equal to L times cosine of 90 degrees. Cosine 90 degrees is 0. So the coordinate along the x-axis will be 0, and YE is equal to L times sine over 90 degrees. Sine of 90 degrees is 1, so this is equal to L.

Which is telling us that our robot will be 0 coordinates units in the x direction and 90 coordinates in the y direction. And if will keep on changing the angle alpha, we observe that this robot will trace a circle. You can practice and find out the coordinates when alpha has any values from 0 to 360 degrees.