Quantum Mechanics & Quantum Computation

Umesh V. Vazirani
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Lecture 7: Quantum Circuits

n qubits

n qubits:

$$\overbrace{0\ 0\ 0\ \cdots\cdots\ 0}^{n}$$

Classical: n-bit string

$$\{0,1\}^n$$

Quantum: $|\psi\rangle = \sum_{x \in \{0,1\}^n} \alpha_x |x\rangle$

$$\alpha_x \in \mathbb{C} \qquad \sum_x |\alpha_x|^2 = 1.$$

Example:

$n = 3$

$$\{0,1\}^3 = \{000, 001, 010, 100, 011, 101, 110, 111\}.$$

$$|\{0,1\}^3| = 2^3 = 8$$

Quantum:

$$|\psi\rangle = \alpha_{000}|000\rangle + \alpha_{001}|001\rangle + \cdots + \cdots + \alpha_{110}|110\rangle + \alpha_{111}|111\rangle.$$

k-level system $\otimes$ l-level system

$$k \rightarrow \sum_{i=0}^{k-1} \alpha_i |i\rangle \qquad \otimes \qquad \sum_{j=0}^{l-1} \alpha_j |j\rangle \leftarrow l$$

$$|\psi\rangle = \sum_{i=0}^{k-1} \sum_{j=0}^{l-1} \gamma_{ij} |ij\rangle$$

γ_{ij}: $k \cdot l$

$|ij\rangle = |i\rangle \otimes |j\rangle$

16 MB 32 MB

48MB

$= 48 \times 10^6$ B.

16×10^6 B $\quad 32 \times 10^6$ B.

$16 \times 32 \times 10^{12}$ B

$\approx 5 \times 10^{14}$ B.

n qubits : $\underbrace{\mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2}_{n} \cong \mathbb{C}^{2^n}$

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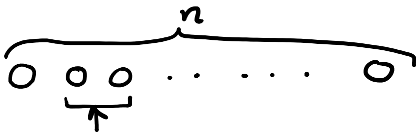
Manipulating n qubits

n qubits:



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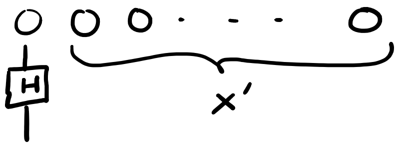


$$\sum_x \alpha_x |x\rangle \xrightarrow{H} \sum_x \beta_x |x\rangle.$$

measure

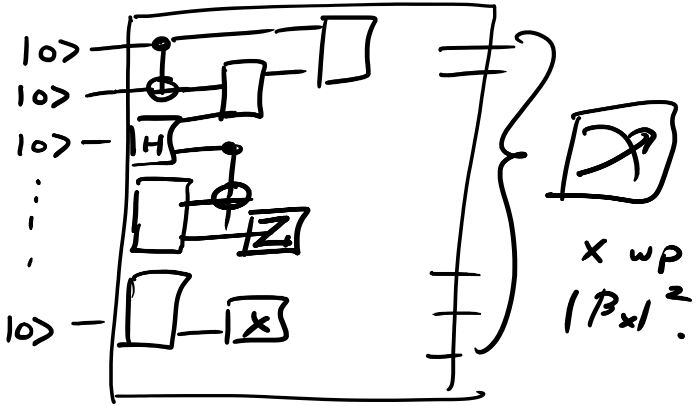
x wp $|\beta_x|^2$.

$$|0x'\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}|0x'\rangle + \frac{1}{\sqrt{2}}|1x'\rangle$$

$$|1x'\rangle \xrightarrow{H} \frac{1}{\sqrt{2}}|0x'\rangle - \frac{1}{\sqrt{2}}|1x'\rangle.$$

$$\left.\begin{aligned} \beta_{0x'} &= \frac{1}{\sqrt{2}}\alpha_{0x'} + \frac{1}{\sqrt{2}}\alpha_{1x'} \\ \beta_{1x'} &= \frac{1}{\sqrt{2}}\alpha_{0x'} - \frac{1}{\sqrt{2}}\alpha_{1x'} \end{aligned}\right\}$$

* Exponential ✓
* Manipulate ✓
* Limited access. ✓



x wp $|\beta_x|^2$.

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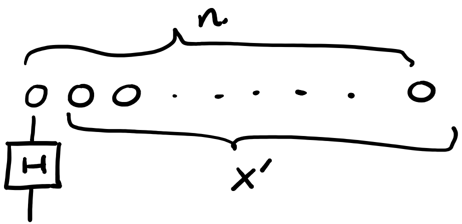
Lecture 7: Quantum Circuits

Universal gate set



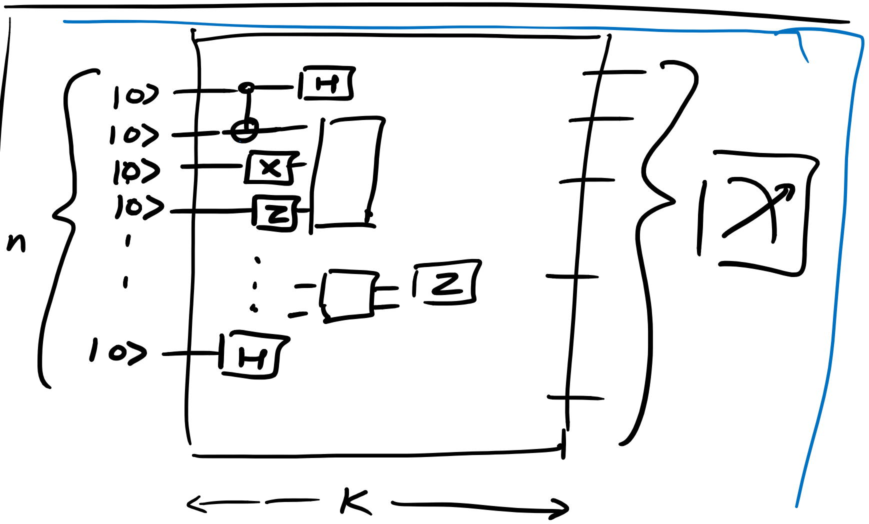
$$\sum_x \alpha_x |x\rangle \xrightarrow{H} \sum_x \beta_x |x\rangle$$

$$|0x'\rangle \longrightarrow \frac{1}{\sqrt{2}}|0x'\rangle + \frac{1}{\sqrt{2}}|1x'\rangle$$

$$|1x'\rangle \longrightarrow \frac{1}{\sqrt{2}}|0x'\rangle - \frac{1}{\sqrt{2}}|1x'\rangle$$

$$\begin{cases} \beta_{0x'} = \frac{1}{\sqrt{2}}\alpha_{0x'} + \frac{1}{\sqrt{2}}\alpha_{1x'} \\ \beta_{1x'} = \frac{1}{\sqrt{2}}\alpha_{0x'} - \frac{1}{\sqrt{2}}\alpha_{1x'} \end{cases}$$

- * Exponential superposition.
- * Gates – update all amplitudes but structured way..
- * Measurement – limited access.



measure: n bit string

$y \quad |\beta_y|^2$

- one & two-qubit gates.

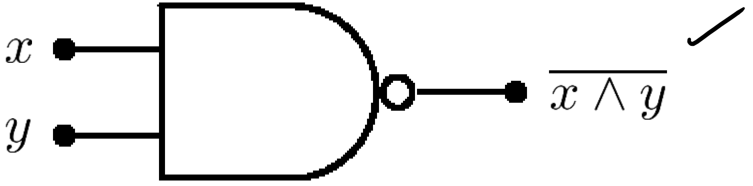
- CNOT, X, Z, H, $\pi/8$

- CSWAP, H, ...

Universal gate set

- In classical circuits, a certain set of gates enables universal computation.

- Ex) NAND is universal



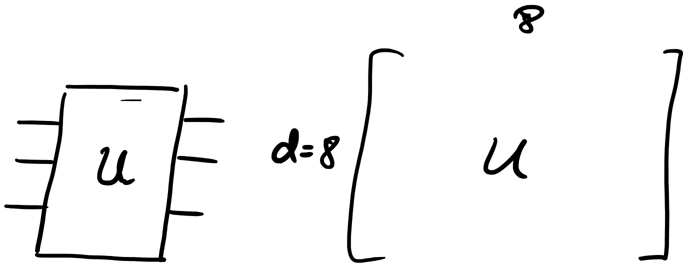
$x = y = 1$ output $= 0$

Universal quantum gate set

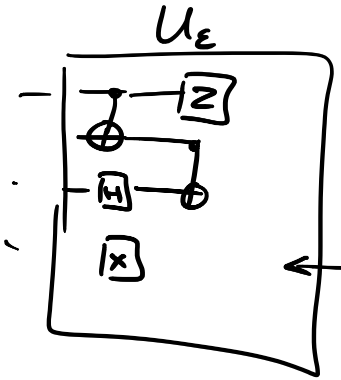
- Quantum analogue
  - CNOT, H, X, Z, $\frac{\pi}{8}$ rotations
  - CSWAP, H, ...

Universal quantum gate set

- Quantum analogue
  - CNOT, H, X, Z, $\frac{\pi}{8}$ rotations



- What does it mean?
  - Clearly, we cannot implement an arbitrary U with infinite precision.
  - Instead, given ϵ, we implement U_ϵ which is ϵ-close to U.



$$|U_\varepsilon - U| \leq \varepsilon$$

$$\# \text{gates} = O\left(d^2 \log^3 \frac{1}{\varepsilon}\right)$$

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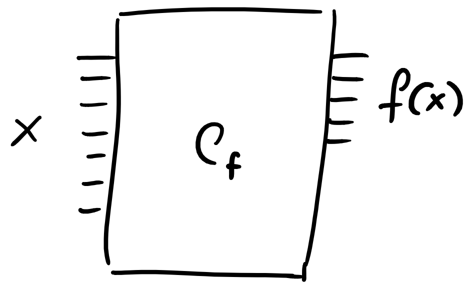
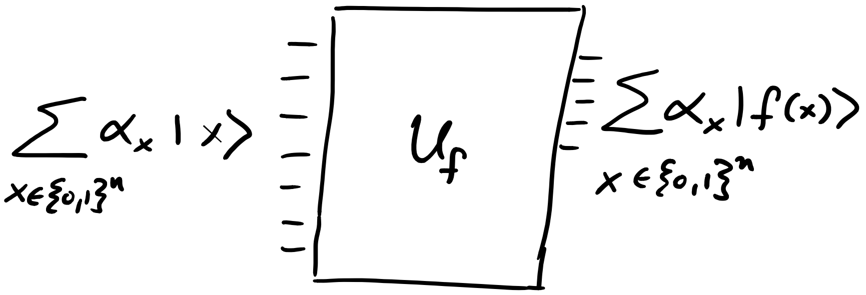
Reversible computation

$$f: \{0,1\}^n \to \{0,1\}^m$$



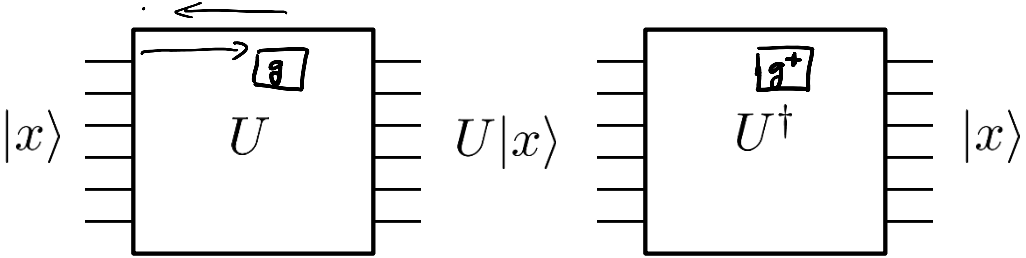


Unitary.

Reversible computation

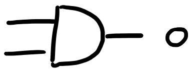
- Quantum computers are reversible.

- Why?



$f: \{0,1\}^n \to \{0,1\}^m$

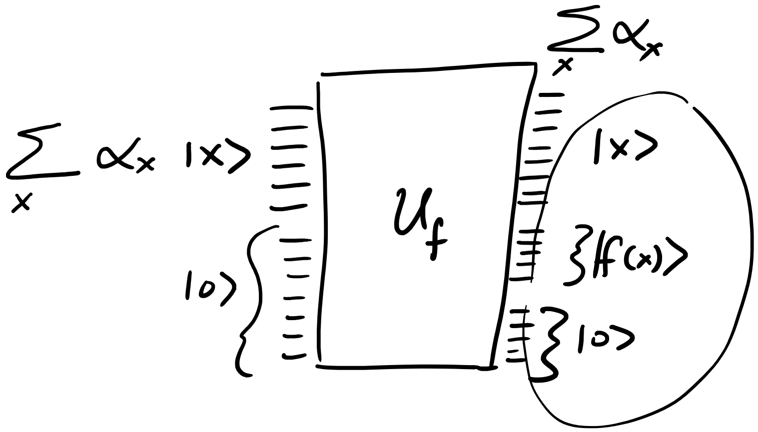
$m \leq n$



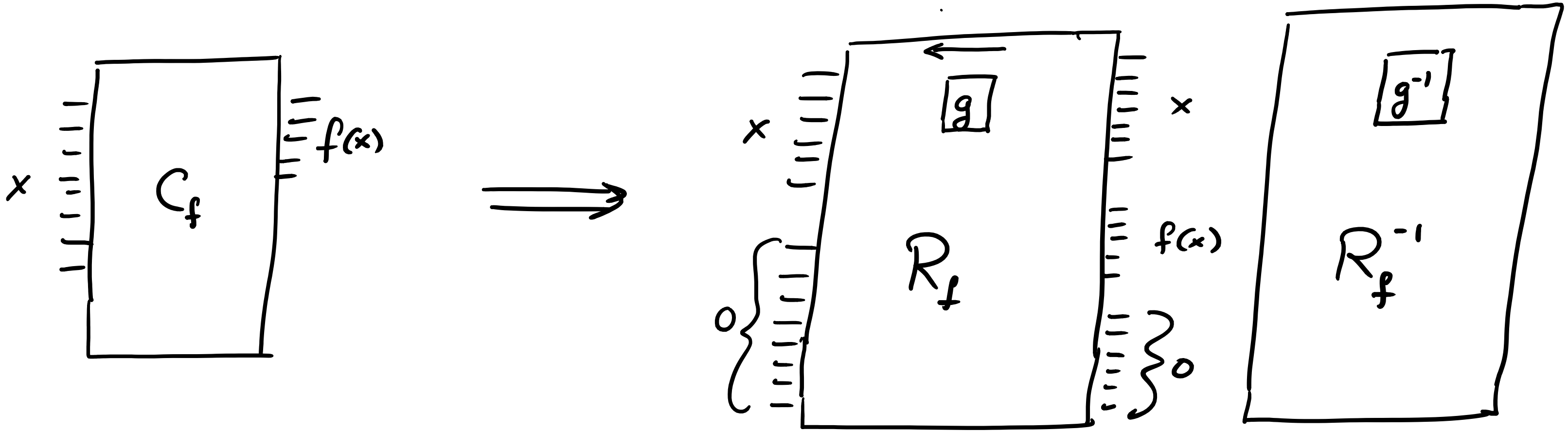
Benioff

Benett Landauer

Reversible computation



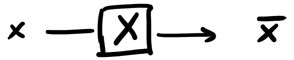
∑ₓ αₓ |x⟩
|0⟩
U_f
∑ₓ αₓ
|x⟩
|f(x)⟩
|0⟩



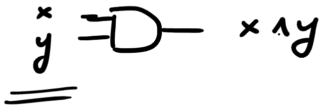
x
C_f
f(x)
x
g
R_f
0
x
f(x)
0
g⁻¹
R_f⁻¹

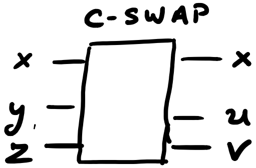
Classical circuits: NAND gate is universal.

AND, NOT.



$0 \to 1$
$1 \to 0$



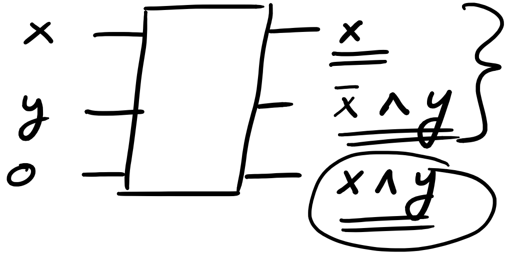


if $x = 1$ then swap $y \leftrightarrow z$.

if $x = 0$ $u = y$ & $v = z$.

$x = 1$ $u = z$ & $v = y$.

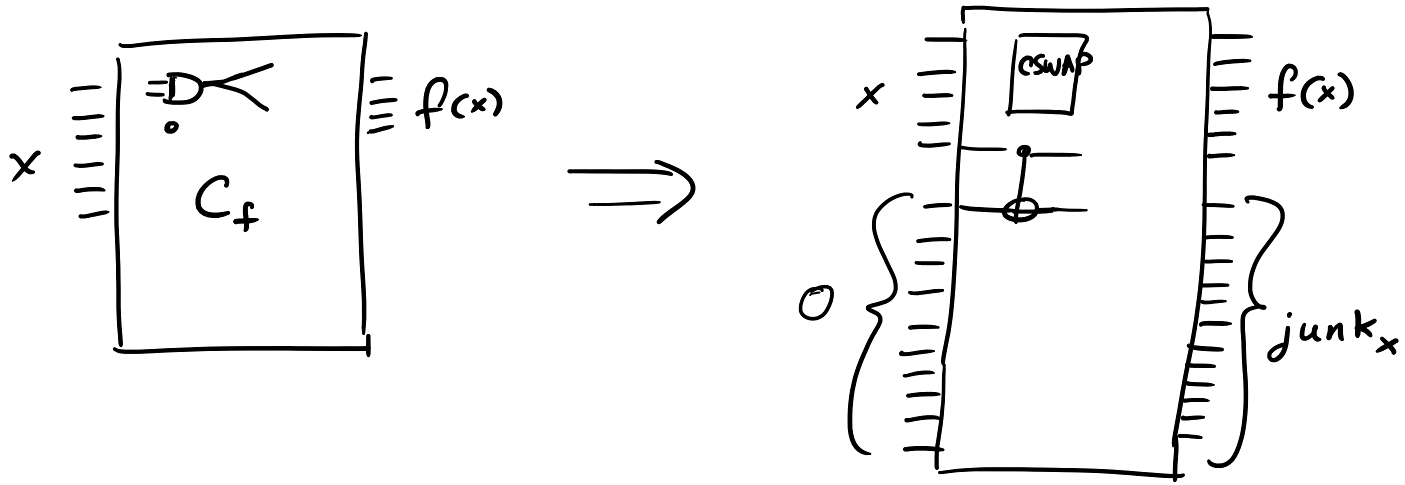
NOT, CSWAP, CNOT

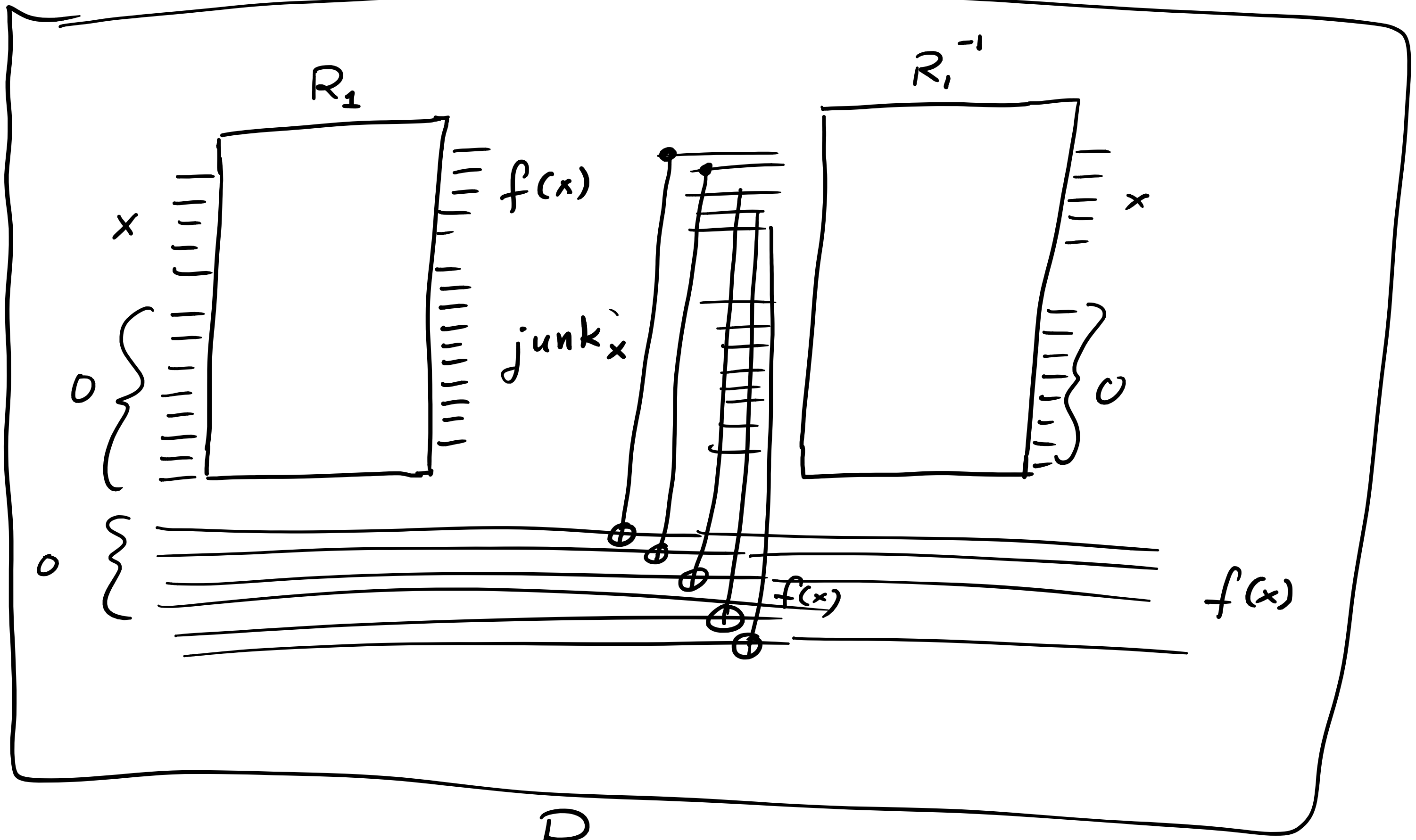


$CSWAP^2 = I$.

$x = 0$ output $= 0$

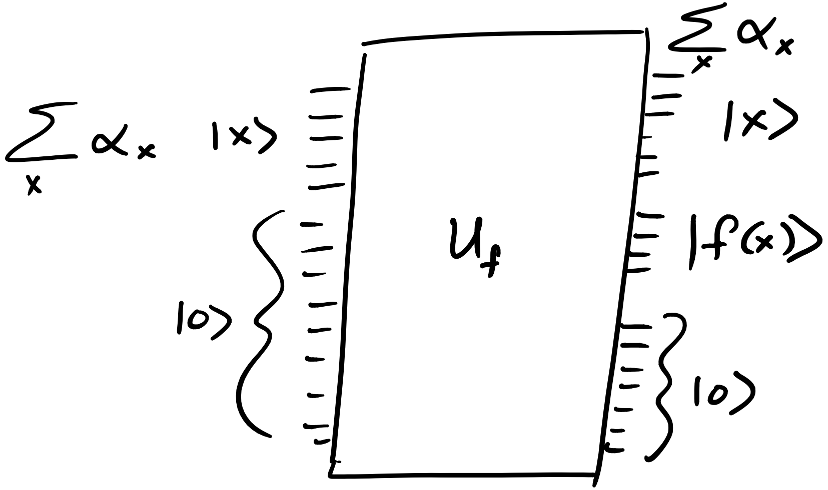
$x = 1$ output $= y$


x
C_f
f(x)
x
CSWAP
f(x)
0
$junk_x$



R_f

CSWAP, CNOT, X quantum gates.



Interference

$f: \{0,1\} \to \{0,1\}$ $f(x) = x.$