Systems of Linear Equations

PART 2: SOLVING LINEAR EQUATIONS USING MATRIX ALGEBRA

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Solving Linear Equations

- Let’s find the price of Apples and Oranges when

3 Apples plus 5 Oranges cost 1.70

5 Apples plus 1 Orange cost 1.00

How much does each Apple cost?

How much does each Orange cost?

Four Techniques

- ✓ Substitution
- ✓ Graphical
- ➢ Matrix Algebra
- Cramer's Rule

Solving Linear Equations with Matrix Algebra (1)

3 Apples plus 5 Oranges cost 1.70
5 Apples plus 1 Orange cost 1.00

In Matrix Form this becomes

$$\begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.70 \\ 1.00 \end{bmatrix}$$

Now let's look at how to understand and work with a matrix

Solving Linear Equations with Matrix Algebra (2)

A matrix problem can be defined as $\boldsymbol{A}x = b$ where

A is a 2 x 2 matrix $\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix}$

x is a 2 x 1 column vector with the unknowns $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

b is a 2 x 1 coefficient matrix $b = \begin{bmatrix} 1.70 \\ 1.00 \end{bmatrix}$

Solving Linear Equations with Matrix Algebra (3)

$x = \frac{b}{A}$

Given $\boldsymbol{A}x = b$

The solution to this matrix is calculated using

$$x = \boldsymbol{A}^{-1}b$$

Where A^{-1} is given by

$$A^{-1} = \frac{1}{\Delta}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Calculating the Determinant

$\Delta = |A|$ is the determinant of matrix **A**

$$\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$\Delta = ad - bc$$

Calculating the Determinant (2)

For our matrix

$$\boldsymbol{A} = \begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix}$$

$$\Delta = 3 \times 1 - 5 \times 5$$

$$\Delta = -22$$

Calculate the Inverse of the Matrix

$$A^{-1} = \frac{1}{\Delta}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{-22}\begin{bmatrix} 1 & -5 \\ -5 & 3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{-22} & \frac{-5}{-22} \\ \frac{-5}{-22} & \frac{3}{-22} \end{bmatrix}$$

Solving the Equation (1)

$$x = A^{-1}b$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{-22} & \frac{-5}{-22} \\ \frac{-5}{-22} & \frac{3}{-22} \end{bmatrix} \cdot \begin{bmatrix} 1.70 \\ 1.00 \end{bmatrix}$$

Solving the Equation (2)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{-22} & \frac{-5}{-22} \\ \frac{-5}{-22} & \frac{3}{-22} \end{bmatrix} \cdot \begin{bmatrix} 1.70 \\ 1.00 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{-22} \times 1.70 & \frac{-5}{-22} \times 1.00 \\ \frac{-5}{-22} \times 1.70 & \frac{3}{-22} \times 1.00 \end{bmatrix}$$

Solving the Equation (3)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{-22} \times 1.70 & \frac{-5}{-22} \times 1.00 \\ \frac{-5}{-22} \times 1.70 & \frac{3}{-22} \times 1.00 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -0.07727 & 0.22727 \\ 0.38636 & -0.13636 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -0.07727 + 0.22727 \\ 0.38636 - 0.13636 \end{bmatrix}$$

$$\begin{bmatrix} 0.15 \\ 0.25 \end{bmatrix}$$

Solving the Equation (3)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -0.07727 + 0.22727 \\ 0.38636 - 0.13636 \end{bmatrix}$$

$$\begin{bmatrix} 0.15 \\ 0.25 \end{bmatrix}$$

$$(x_1, x_2) = (0.15, 0.25)$$

Solving Linear Equations with Matrix Algebra (revisited)

3 Apples plus 5 Oranges cost 1.70
5 Apples plus 1 Orange cost 1.00

$$(x_1, x_2) = (0.15, 0.25)$$

Apples cost 0.15 each
Oranges cost 0.25 each

Review of Four Techniques

- ✓ Substitution
- ✓ Graphical
- ✓ Matrix Algebra
- Cramer's Rule

Thank You!