

Systems of Linear Equations

PART 2: SOLVING LINEAR EQUATIONS USING MATRIX ALGEBRA

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Solving Linear Equations

• Let's find the price of Apples and Oranges when

3 Apples plus 5 Oranges cost 1.705 Apples plus 1 Orange cost 1.00

How much does each Apple cost? How much does each Orange cost?

Four Techniques

✓ Substitution
 ✓ Graphical
 ➢ Matrix Algebra

• Cramer's Rule

Solving Linear Equations with Matrix Algebra (1)

3 Apples plus 5 Oranges cost 1.70 5 Apples plus 1 Orange cost 1.00

In Matrix Form this becomes

 $\begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.70 \\ 1.00 \end{bmatrix}$

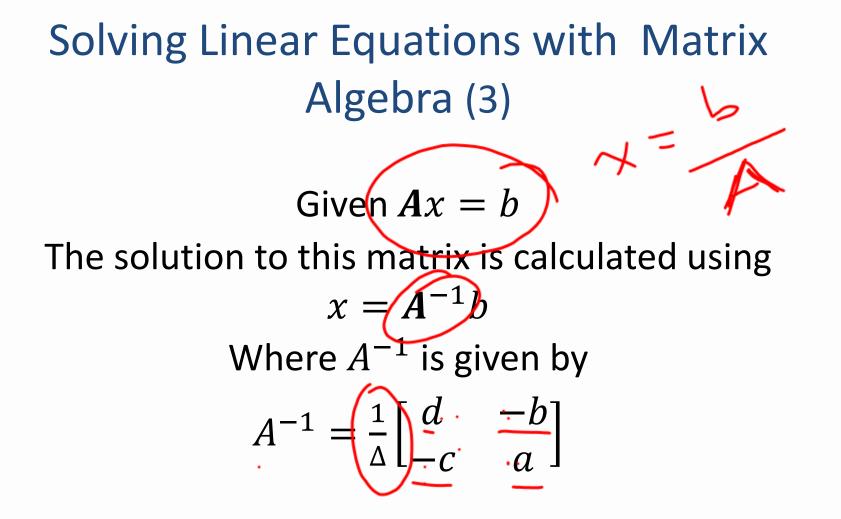
Now let's look at how to understand and work with a matrix Solving Linear Equations with Matrix Algebra (2)

 $b = \begin{bmatrix} 1.70 \\ 1.00 \end{bmatrix}$

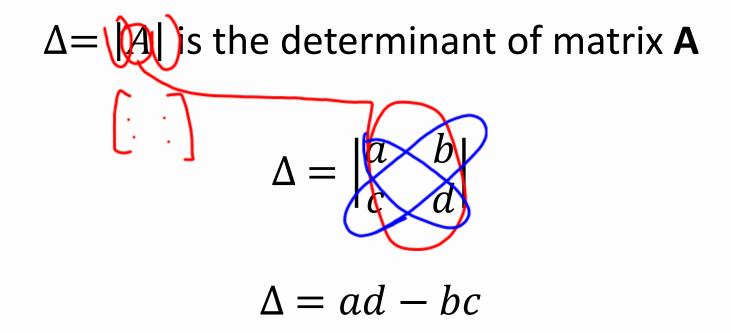
A matrix problem can be defined as Ax = b where

A is a 2 x 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix}$ *x* is a 2 x 1 column vector with the unknowns $x = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$

b is a 2 x 1 coefficient matrix

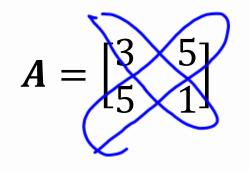


Calculating the Determinant



Calculating the Determinant (2)

For our matrix

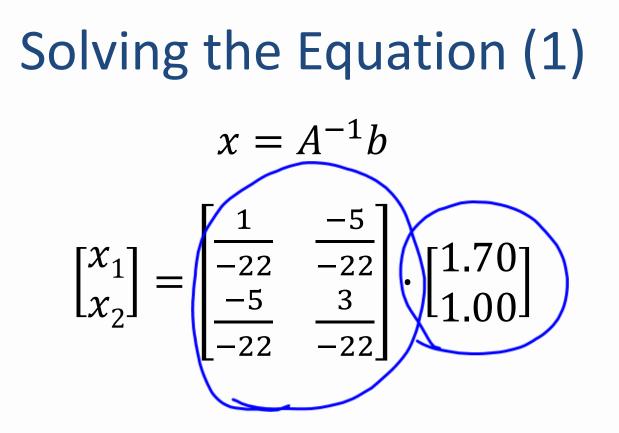


 $\Delta = 3 \times 1 - 5 \times 5$

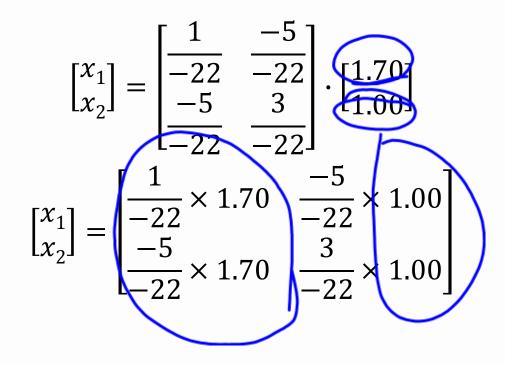
 $\Delta = -22$

Calculate the Inverse of the Matrix

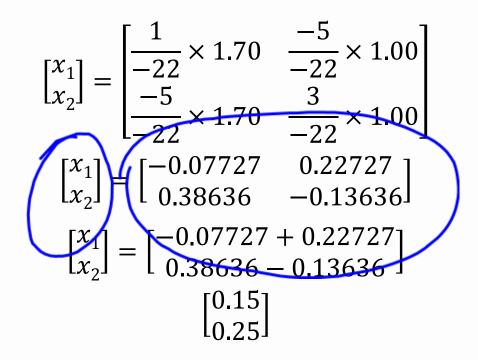
$A^{-1} =$	$\frac{1}{\Delta} \begin{bmatrix} d \\ -c \end{bmatrix}$	$\begin{bmatrix} -b\\ a \end{bmatrix}$
	$\frac{1}{22} \begin{bmatrix} 1 \\ -5 \end{bmatrix}$	· -5 3]
$A^{-1} =$	1 -22 -5 -22	$ \begin{array}{c} -5\\ -22\\ 3\\ -22 \end{array} $



Solving the Equation (2)



Solving the Equation (3)



Solving the Equation (3) $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -0.07727 + 0.22727 \\ 0.38636 - 0.13636 \end{bmatrix}$ $\begin{bmatrix} 0.15 \\ 0.25 \end{bmatrix}$ $(x_1, x_2) = (0.15, 0.25)$

Solving Linear Equations with Matrix Algebra (revisited)

> 3 Apples plus 5 Oranges cost 1.70 5 Apples plus 1 Orange cost 1.00

 $(x_1, x_2) = (0.15, 0.25)$

Apples cost 0.15 each Oranges cost 0.25 each

Review of Four Techniques

✓ Substitution
✓ Graphical
✓ Matrix Algebra

• Cramer's Rule

Thank You!



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