Systems of Linear Equations

PART 3: CRAMER’S RULE

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Solving Linear Equations

- Let’s find the price of Apples and Oranges when

3 Apples plus 5 Oranges cost 1.70
5 Apples plus 1 Orange   cost 1.00

How much does each Apple cost?
How much does each Orange cost?

Four Techniques

- ✓ Substitution
- ✓ Graphical
- ✓ Matrix Algebra
- ➢ Cramer's Rule

On to our problem:
Solving a Linear Equation

3 Apples plus 5 Oranges cost 1.70

5 Apples plus 1 Orange cost 1.00

Cramer's Rule

Starts with a system of equations

$$\boldsymbol{Ax} = b$$

The solution is calculated using

$$x_1 = \frac{|A_1|}{|A|}$$

$$x_1 = \frac{|A_1|}{|A|}, x_2 = \frac{|A_2|}{|A|}, \ldots, x_i = \frac{|A_i|}{|A|}$$

Cramer's Rule: Calculating A_i

$$x_i = \frac{\Delta A_i}{\Delta A} = \frac{|A_i|}{|A|}$$

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= b_1 \\ a_{21}x_1 + a_{22}x_2 &= b_2 \end{aligned}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

Cramer's Rule: Calculating ΔA_i

$$\Delta \boldsymbol{A} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$\Delta \boldsymbol{A}_1 = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}$$

$$\Delta \boldsymbol{A}_2 = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$$

Cramer's Rule: Calculating x_i

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{\Delta A_1}{\Delta A}$$

$$x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} \quad \frac{\Delta A_2}{\Delta A}$$

Solving for x_1, x_2

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$x_1 = \frac{a_{22}b_1 - a_{12}b_2}{a_{11}a_{22} - a_{12}a_{21}}$$

Solving for x_1, x_2

$$x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}}$$

$$x_2 = \frac{a_{11}b_2 - a_{21}b_1}{a_{11}a_{22} - a_{12}a_{21}}$$

Back to Apples and Oranges

$$\begin{bmatrix} 3 & 5 \\ 5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.70 \\ 1.00 \end{bmatrix}$$

$$x_1 = \frac{\begin{vmatrix} 1.7 & 5 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 5 \\ 5 & 1 \end{vmatrix}} = \frac{1.7 \times 1 - 1 \times 5}{3 \times 1 - 5 \times 5}$$

$$x_2 = \frac{\begin{vmatrix} 3 & 1.7 \\ 5 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & 5 \\ 5 & 1 \end{vmatrix}} = \frac{3 \times 1 - 5 \times 1.7}{3 \times 1 - 5 \times 5}$$

$$x_1 = \frac{-3.3}{-22} = 0.15$$

$$x_2 = \frac{-5.5}{-22} = 0.25$$

Four Techniques for Solving Linear Equations

- ✓ Substitution
- ✓ Graphical
- ✓ Matrix Algebra
- ✓ Cramer's Rule

Thank You!