Facility Location Models
Agenda

• Continuous Single Facility Problems
  ■ Weber or Minimum Distance
  ■ Center of Gravity

• Network Facility Location Problems
  ■ Mixed Integer Linear Program (MILP) Models
  ■ Solve MILPs in spreadsheets

• Multiple Location Selection Models
  ■ Trade-off between costs and level of service (LOS)
  ■ Solve in Spreadsheets

• Sensitivity Analysis with MILP models
Continuous Location Models
Continuous Location Problems

- **Problem Objective (Single Location Weber Problem)**
  - Find the single location that minimizes the transportation costs from this point to K destination points.

- **Assumptions**
  - K points within Euclidean space
  - Center location can be anywhere on the space
  - Transportation cost is the distance multiplied by volume (or weight)

Mathematically:

\[
\text{Min} \quad z = \sum_{k \in K} w_k \cdot d_k (x, y) = \sum_{k \in K} w_k \sqrt{(x - x_k)^2 + (y - y_k)^2}
\]

**Index**

- locations $k$

**Input Data**

- $w_k =$ Weight of location $k \quad \forall k \in K$
- $x_k =$ Horizontal or X coordinate of location $k \quad \forall k \in K$
- $y_k =$ Vertical or Y coordinate of location $k \quad \forall k \in K$
- $d_k =$ Distance from location $k$ to centerpoint $(x, y) \quad \forall k \in K$

**Decision Variables**

- $x =$ Horizontal or X coordinate of center point
- $y =$ Vertical or Y coordinate of center point
NERD serves the highly profitable quality root beer soda market and currently sell their product in just three cities: Boston, Springfield, and Providence. Marketing has estimated potential root beer consumption for each market.

Where should NERD set up its distribution center (DC) to serve these three cities in order to minimize expected transportation costs?

<table>
<thead>
<tr>
<th>ID</th>
<th>City Name</th>
<th>Wgt</th>
</tr>
</thead>
<tbody>
<tr>
<td>BO</td>
<td>Boston</td>
<td>425</td>
</tr>
<tr>
<td>PR</td>
<td>Providence</td>
<td>320</td>
</tr>
<tr>
<td>SP</td>
<td>Springfield</td>
<td>220</td>
</tr>
</tbody>
</table>
**New England Root Beer Distributors (NERD)**

\[
\text{Min} \quad z = \sum_{k \in K} w_k d_k(x, y) = \sum_{k \in K} w_k \sqrt{(x - x_k)^2 + (y - y_k)^2}
\]

<table>
<thead>
<tr>
<th>ID</th>
<th>City Name</th>
<th>Wgt</th>
<th>(x_k)</th>
<th>(y_k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BO</td>
<td>Boston</td>
<td>425</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>PR</td>
<td>Providence</td>
<td>320</td>
<td>86</td>
<td>40</td>
</tr>
<tr>
<td>SP</td>
<td>Springfield</td>
<td>220</td>
<td>20</td>
<td>60</td>
</tr>
</tbody>
</table>

\[=\text{SQRT}((C2-S\!S6)^2+(D2-S\!D6)^2)\]

\[=B2\times E2\]

\[=\text{SUM}(F2:F4)\]
Comparing to Center of Gravity

- **CoG** is an intuitive approach based on physics
  - Find the average of the weighted X and Y coordinates
  - Finds the centroid of the shape – point of balance

\[
x = \left(\frac{425}{965}\right)(100) + \left(\frac{320}{965}\right)(86) + \left(\frac{220}{965}\right)(20) \approx 77
\]

\[
y = \left(\frac{425}{965}\right)(80) + \left(\frac{320}{965}\right)(40) + \left(\frac{220}{965}\right)(60) \approx 62
\]

- Why are they different? And which should we use?
- **CoG** does NOT find the minimum weighted distance, it minimizes the weighted-squared distance!
New England Root Beer Distributors (NERD)

• Yes, this location minimizes total weighted Euclidean distance, but . . .
  ■ We are not living on a Euclidean plane!
  ■ Every location is not feasible nor equally desirable
  ■ There is an underlying network of highways, rail lines, ports, etc.
• So, most analysis selects optimal locations from a candidate list

<table>
<thead>
<tr>
<th>ID</th>
<th>City Name</th>
<th>Wgt</th>
</tr>
</thead>
<tbody>
<tr>
<td>BO</td>
<td>Boston</td>
<td>425</td>
</tr>
<tr>
<td>PR</td>
<td>Providence</td>
<td>320</td>
</tr>
<tr>
<td>SP</td>
<td>Springfield</td>
<td>220</td>
</tr>
</tbody>
</table>
Network Facility Location Problem
Network Facility Location Problem

- **Problem Objective**
  - Find the location(s) from a list of candidates that minimizes the total costs of delivering from this point to \( n \) destination points over a given network.

Indices
- Distribution Centers \( i \)
- Customers \( j \)

Input Data
- \( S_i = \text{Available supply at DC } i \text{ (units)} \quad \forall i \in S \)
- \( D_j = \text{Demand by Customer } j \text{ (units)} \quad \forall j \in D \)
- \( c_{ij} = \text{Cost to serve Customer } j \text{ from DC } i \text{ ($/unit)} \quad \forall i, j \)
- \( f_i = \text{Fixed cost for opening DC } i \text{ ($)} \quad \forall i \in S \)
- \( P_{\text{MIN}} = \text{Minimum number of DCs required to open} \)
- \( P_{\text{MAX}} = \text{Maximum number of DCs allowed to open} \)
- \( M = \text{A really big number, but not too big!} \)

Decision Variables
- \( x_{ij} = \text{Flow on arc from DC } i \text{ to Customer } j \text{ (units)} \quad \forall i, j \)
- \( Y_i = \{0, 1\} \quad \forall i \)

Objectives
- Minimize total costs: \( z = \sum_i \sum_j c_{ij} x_{ij} + \sum_i f_i Y_i \)

Subject to
- \( \sum_j x_{ij} \leq S_i \quad \forall i \in S \)
- \( \sum_j x_{ij} \geq D_j \quad \forall j \in D \)
- \( x_{ij} - M_i Y_i \leq 0 \quad \forall ij \)
- \( \sum_i Y_i \geq P_{\text{MIN}} \)
- \( \sum_i Y_i \leq P_{\text{MAX}} \)
- \( x_{ij} \geq 0 \quad \forall ij \)
- \( Y_i = \{0, 1\} \quad \forall i \)

Same as the Transportation Problem formulation

Fixed cost of opening and operating DCs

Logical or linking constraint between \( x_{ij} \)'s and \( Y_i \)'s

Minimum and maximum number of DCs allowed to open

Declaring \( Y_i \) is a Binary variable
Logical or Linking Constraint

Min \[ z = \sum_i \sum_j c_{ij} x_{ij} + \sum_i f_i Y_i \]

s.t.

\[ \sum_j x_{ij} \leq S_i \quad \forall i \in S \]
\[ \sum_i x_{ij} \geq D_j \quad \forall j \in D \]
\[ x_{ij} - M_i Y_i \leq 0 \quad \forall ij \]
\[ \sum_i Y_i \geq P_{\text{MIN}} \]
\[ \sum_i Y_i \leq P_{\text{MAX}} \]
\[ x_{ij} \geq 0 \quad \forall ij \]
\[ Y_i = \{0, 1\} \quad \forall i \]

• What does this constraint mean?
  - It simply says, we cannot deliver from a DC unless we open that DC!

• What if it was not included?
  - No DCs would be opened – it becomes the transportation problem!

• How does it work?
  - if \( x_{ij} = 0 \), then \( Y_i \) can = 0 or 1 (min will drive to 0)
  - if \( x_{ij} \geq 1 \), then \( Y_i \) must = 1 (or else constraint is violated)

• What is the best value for \( M_{ij} \)?
  - You should set \( M_{ij} \geq \) maximum value of \( x_{ij} \)
  - It is perfectly acceptable to set them all to one value, such as \( \Sigma D_j \)

Input Data

\( S_i = \) Available supply at DC \( i \) (units) \( \forall i \in S \)
\( D_j = \) Demand by Customer \( j \) (units) \( \forall j \in D \)
\( c_{ij} = \) Cost to serve Customer \( j \) from DC \( i \) ($/unit) \( \forall i, j \)
\( f_i = \) Fixed cost for opening DC \( i \) ($) \( \forall i \in S \)
\( P_{\text{MIN}} = \) Minimum number of DCs required to open
\( P_{\text{MAX}} = \) Maximum number of DCs allowed to open
\( M = \) A really big number, but not too big!

Indices

Distribution Centers \( i \)
Customers \( j \)

Decision Variables

\( x_{ij} = \) Flow on arc from DC \( i \) to Customer \( j \) (units) \( \forall i, j \)
\( Y_i = 1 \) if DC \( i \) is opened; =0 otherwise \( \forall i \in S \)
Solving the Network Facility Location Problem in Spreadsheet
NERD has expanded its market to 12 cities across New England. They have selected 5 candidate DC locations: BO, NA, PR, SP, and WO.

Where should they locate their DC?

<table>
<thead>
<tr>
<th>ID</th>
<th>City Name</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>BO</td>
<td>Boston</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>BR</td>
<td>Brattleboro</td>
<td>15</td>
<td>119</td>
</tr>
<tr>
<td>CO</td>
<td>Concord</td>
<td>76</td>
<td>145</td>
</tr>
<tr>
<td>HA</td>
<td>Hartford</td>
<td>13</td>
<td>33</td>
</tr>
<tr>
<td>MN</td>
<td>Manchester</td>
<td>80</td>
<td>132</td>
</tr>
<tr>
<td>NA</td>
<td>Nashua</td>
<td>80</td>
<td>112</td>
</tr>
<tr>
<td>NH</td>
<td>New Haven</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>NL</td>
<td>New London</td>
<td>40</td>
<td>6</td>
</tr>
<tr>
<td>PO</td>
<td>Portsmouth</td>
<td>120</td>
<td>139</td>
</tr>
<tr>
<td>PR</td>
<td>Providence</td>
<td>86</td>
<td>40</td>
</tr>
<tr>
<td>SP</td>
<td>Springfield</td>
<td>20</td>
<td>60</td>
</tr>
<tr>
<td>WO</td>
<td>Worcester</td>
<td>66</td>
<td>73</td>
</tr>
</tbody>
</table>

For fun, where would you locate the DC, assuming continuous location?
NERD has expanded its market to 12 cities across New England. They have selected 5 candidate DC locations: BO, NA, PR, SP, and WO.

Where should they locate their DC?

For simplicity, assume:

- \( f_i = \$10,000 \)
- \( c_{ij} = (1 \ \$/\text{mile})(d_{ij}) \)
- \( S_i = \Sigma D_j \)
- \( P_{\text{MIN}}=P_{\text{MAX}} = 1 \)

### Distance Matrix \( d_{ij} \) (miles)

<table>
<thead>
<tr>
<th>ID</th>
<th>City</th>
<th>( D_j )</th>
<th>BO</th>
<th>NA</th>
<th>PR</th>
<th>SP</th>
<th>WO</th>
</tr>
</thead>
<tbody>
<tr>
<td>BO</td>
<td>Boston</td>
<td>425</td>
<td>0</td>
<td>37</td>
<td>42</td>
<td>82</td>
<td>34</td>
</tr>
<tr>
<td>BR</td>
<td>Brattleboro</td>
<td>12</td>
<td>93</td>
<td>65</td>
<td>106</td>
<td>59</td>
<td>68</td>
</tr>
<tr>
<td>CO</td>
<td>Concord</td>
<td>43</td>
<td>69</td>
<td>98</td>
<td>103</td>
<td>105</td>
<td>72</td>
</tr>
<tr>
<td>HA</td>
<td>Hartford</td>
<td>720</td>
<td>98</td>
<td>103</td>
<td>73</td>
<td>27</td>
<td>66</td>
</tr>
<tr>
<td>MN</td>
<td>Manchester</td>
<td>110</td>
<td>55</td>
<td>20</td>
<td>92</td>
<td>93</td>
<td>60</td>
</tr>
<tr>
<td>NA</td>
<td>Nashua</td>
<td>86</td>
<td>37</td>
<td>72</td>
<td>94</td>
<td>63</td>
<td>98</td>
</tr>
<tr>
<td>NH</td>
<td>New Haven</td>
<td>129</td>
<td>128</td>
<td>137</td>
<td>94</td>
<td>63</td>
<td>98</td>
</tr>
<tr>
<td>NL</td>
<td>New London</td>
<td>28</td>
<td>95</td>
<td>48</td>
<td>9</td>
<td>68</td>
<td>38</td>
</tr>
<tr>
<td>PO</td>
<td>Portsmouth</td>
<td>66</td>
<td>62</td>
<td>48</td>
<td>104</td>
<td>127</td>
<td>85</td>
</tr>
<tr>
<td>PR</td>
<td>Providence</td>
<td>320</td>
<td>42</td>
<td>72</td>
<td>0</td>
<td>68</td>
<td>38</td>
</tr>
<tr>
<td>SP</td>
<td>Springfield</td>
<td>220</td>
<td>82</td>
<td>79</td>
<td>68</td>
<td>0</td>
<td>47</td>
</tr>
<tr>
<td>WO</td>
<td>Worcester</td>
<td>182</td>
<td>34</td>
<td>41</td>
<td>38</td>
<td>47</td>
<td>0</td>
</tr>
</tbody>
</table>
NERD 2 Model in Spreadsheet (yikes!)

- Maximum supply at each DC, $\sum_{i} x_{ij} \leq S_{i} \quad \forall i \in S$
- Minimum demand at each Customer, $\sum_{j} x_{ij} \geq D_{j} \quad \forall j \in D$
- Minimum & Maximum Number of DCs to open, $\sum_{i} Y_{i} \geq P_{\text{MIN}}$ $\sum_{i} Y_{i} \leq P_{\text{MAX}}$
- Linking constraints for each $x_{ij}$ and $Y_{i}$ combination, $x_{ij} - M_{ij}Y_{i} \leq 0 \quad \forall ij$
Solving NERD 2 Model in Spreadsheet

Solver Parameters:
- Set Objective: $SBS31$
- To: Max
- By Changing Variable Cells: $SC35:SB055$
- Subject to the Constraints:
  - $SBPS10:SBPS14 <= $BR$310:$BR$314$
  - $SBPS17:SBPS28 >= $BR$317:$BR$328$
  - $SBPS31 <= $BR$331$
  - $SBPS32 <= $BR$332$
  - $SBPS35:SBPS94 <= $BR$335:$BR$394$
  - $SC35:SC35 = binary$

Make Unconstrained Variables Non-Negative
Select a Solving Method: Simplex LP

Solving Method
Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.
Solving NERD 2 Model in Spreadsheet

Solution:
- Open DC in Worcester (WO)
- Distribute to all cities from WO

For a total cost of $89,478 per week.

But, is opening just one DC the right number? Should we open more?
Selecting Multiple Facilities
Trade-Offs for Multiple Locations

Supply Chain Design is all about finding balance between:

- **Costs**
  - Inventory Costs
  - Facility Costs
  - Transportation Costs
  - Operating (Labor) Costs
  - Asset Costs

- **Service**
  - Customer Satisfaction
  - Sales Growth
  - Increased Market Share
  - Response Times
  - Level of Service

Find the right balance that improves profitability and customer satisfaction

Source: Steve Ellet, Chainalytics
Cost Impacts

Number of DCs

Costs ($)

Warehousing Costs

Inventory Costs

Transportation Costs
- Inbound
- Outbound Delivery

Total Costs

Source: Steve Ellet, Chainalytics

\[ SS_{NEW} = SS_{OLD} \sqrt{\frac{N_{NEW}}{N_{OLD}}} \]
Level of Service Impacts

• What is Level of Service (LOS)?
  • Tactical Inventory Replenishment Policies
    – Cycle Service Level (CSL)
    – Item Fill Rate (IFR)
    – Cost of Stock Out Event (CSOE)
    – Cost of Item Short (CIS)
  • Do these make sense for strategic network design?
    – No – these are more influenced by the stocking level, not the location of facilities!
  • Strategic Network Design
    – Distance to customer is used as a proxy for LOS
    – Provides “opportunity” to meet tactical service objectives
Level of Service – Distance to Customers

• How do we measure this?
  ■ Allowable distance
    ◇ No customer may be more than X miles from a DC
    ◇ Y% of all customers need to be within X miles of a DC
  ■ Average distance
    ◇ Average distance for customers to DC must be less than X miles
    ◇ Weighted average distance for customers to a DC must be less than X miles

• How do we model this?
  ■ Report distance results from each run
  ■ Add constraints to enforce
  ■ Change the objective function
Selecting Optimal DCs
NERD2 Problem – Single DC Solution

Single DC Location: WO
Total Cost: $89,478 per week

Average Customer Distance (miles) 56.67
Average Demand Distance (miles) 45.52
Percent Demand within distance bracket:
- distance ≤ 25 miles 10%
- 25 miles < distance ≤ 50 miles 60%
- 50 miles < distance ≤ 75 miles 18%
- 75 miles < distance ≤ 100 miles 11%
- distance > 100 miles 0%

Suppose that they want to open 2 DCs. Where should they locate them?

(Resolve the problem with $P_{\text{MIN}}=P_{\text{MAX}}=2$)
NERD2 Problem – Two DC Solution (\(P_{\text{MIN}}=P_{\text{MAX}}=2\))

2 DC Locations: SP & BO
- Boston (BO) DC (1232 units) serves: BO, PO, CO, MN, NA, WO, PR
- Springfield (SP) DC (514 units) serves: BR, SP, HA, NH, NL

Total Cost: $69,725 per week

Average Customer Distance (miles): 42.08
Average Demand Distance (miles): 28.48

Percent Demand within distance bracket:
- distance ≤ 25 miles: 37%
- 25 miles < distance ≤ 50 miles: 41%
- 50 miles < distance ≤ 75 miles: 22%
- 75 miles < distance ≤ 100 miles: 0%
- distance > 100 miles: 0%
NERD2 Problem – Three DC Solution \( (P_{\text{MIN}}=P_{\text{MAX}}=3) \)

3 DC Locations: SP, PR, & BO
- Boston (BO) DC (912 units) serves: BO, PO, CO, MN, NA, WO
- Springfield (SP) DC (486 units) serves: BR, SP, HA, NH
- Providence (PR) DC (348) serves: PR, NL

Total Cost: $66,285 per week

<table>
<thead>
<tr>
<th>Percent Demand within distance bracket:</th>
<th>55%</th>
<th>23%</th>
<th>22%</th>
<th>0%</th>
<th>0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance ≤ 25 miles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25 miles &lt; distance ≤ 50 miles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 miles &lt; distance ≤ 75 miles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>75 miles &lt; distance ≤ 100 miles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>distance&gt; 100 miles</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Average Customer Distance (miles) | 38.58 |
Average Demand Distance (miles) | 20.78 |
NERD2 Problem – Four DC Solution  \[ (P_{\text{MIN}}=P_{\text{MAX}}=4) \]

4 DC Locations: SP, PR, NA, & BO
- Boston (BO) DC (607 units) serves: BO & WO
- Springfield (SP) DC (486 units) serves: BR, SP, HA, NH
- Providence (PR) DC (348) serves: PR & NL
- Nashua (NA) DC (305) serves: CO, PO, MN, NA

Total Cost: $66,781 per week

<table>
<thead>
<tr>
<th>Average Customer Distance (miles)</th>
<th>38.58</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Demand Distance (miles)</td>
<td>20.78</td>
</tr>
</tbody>
</table>

Percent Demand within distance bracket:

<table>
<thead>
<tr>
<th>Distance Bracket</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance ≤ 25 miles</td>
<td>66%</td>
</tr>
<tr>
<td>25 miles &lt; distance ≤ 50 miles</td>
<td>24%</td>
</tr>
<tr>
<td>50 miles &lt; distance ≤ 75 miles</td>
<td>10%</td>
</tr>
<tr>
<td>75 miles &lt; distance ≤ 100 miles</td>
<td>0%</td>
</tr>
<tr>
<td>distance &gt; 100 miles</td>
<td>0%</td>
</tr>
</tbody>
</table>
NERD2 Problem – Five DC Solution (\(P_{\text{MIN}}=P_{\text{MAX}}=5\))

5 DC Locations: SP, PR, NA, WO, & BO
- Boston (BO) DC (425 units) serves: BO
- Springfield (SP) DC (486 units) serves: BR, SP, HA, NH
- Providence (PR) DC (348) serves: PR, NL
- Nashua (NA) DC (305) serves: CO, PO, MN, NA
- Worcester (WO) DC (182) serves: WO

Total Cost: $70,593 per week

### Average Customer Distance (miles)
38.58

### Average Demand Distance (miles)
20.78

**Percent Demand within distance bracket:**

<table>
<thead>
<tr>
<th>Distance Bracket</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance ≤ 25 miles</td>
<td>77%</td>
</tr>
<tr>
<td>25 miles &lt; distance ≤ 50 miles</td>
<td>13%</td>
</tr>
<tr>
<td>50 miles &lt; distance ≤ 75 miles</td>
<td>10%</td>
</tr>
<tr>
<td>75 miles &lt; distance ≤ 100 miles</td>
<td>0%</td>
</tr>
<tr>
<td>distance &gt; 100 miles</td>
<td>0%</td>
</tr>
</tbody>
</table>
Comparing the Options

<table>
<thead>
<tr>
<th></th>
<th>P=1</th>
<th>P=2</th>
<th>P=3</th>
<th>P=4</th>
<th>P=5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Total Cost</strong></td>
<td>$89,478</td>
<td>$69,725</td>
<td>$66,285</td>
<td>$66,781</td>
<td>$70,593</td>
</tr>
<tr>
<td>Transportation Costs</td>
<td>$79,478</td>
<td>$49,725</td>
<td>$36,285</td>
<td>$26,781</td>
<td>$20,593</td>
</tr>
<tr>
<td>Facility Costs</td>
<td>$10,000</td>
<td>$20,000</td>
<td>$30,000</td>
<td>$40,000</td>
<td>$50,000</td>
</tr>
</tbody>
</table>

**Average Distance Metrics**

<table>
<thead>
<tr>
<th>Metric</th>
<th>P=1</th>
<th>P=2</th>
<th>P=3</th>
<th>P=4</th>
<th>P=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Customer Distance (miles)</td>
<td>56.67</td>
<td>42.08</td>
<td>38.58</td>
<td>28.42</td>
<td>25.58</td>
</tr>
<tr>
<td>Average Demand Distance (miles)</td>
<td>45.52</td>
<td>28.48</td>
<td>20.78</td>
<td>15.34</td>
<td>11.79</td>
</tr>
<tr>
<td>Maximum Customer Distance (miles)</td>
<td>98 (NH)</td>
<td>69 (CO)</td>
<td>69 (CO)</td>
<td>63 (NH)</td>
<td>63 (NH)</td>
</tr>
</tbody>
</table>

**Percent Demand within distance bracket:**

<table>
<thead>
<tr>
<th>Distance Bracket</th>
<th>P=1</th>
<th>P=2</th>
<th>P=3</th>
<th>P=4</th>
<th>P=5</th>
</tr>
</thead>
<tbody>
<tr>
<td>≤ 25 miles</td>
<td>10%</td>
<td>37%</td>
<td>55%</td>
<td>66%</td>
<td>77%</td>
</tr>
<tr>
<td>25 miles &lt; distance ≤ 50 miles</td>
<td>60%</td>
<td>41%</td>
<td>23%</td>
<td>24%</td>
<td>13%</td>
</tr>
<tr>
<td>50 miles &lt; distance ≤ 75 miles</td>
<td>18%</td>
<td>22%</td>
<td>22%</td>
<td>10%</td>
<td>10%</td>
</tr>
<tr>
<td>75 miles &lt; distance ≤ 100 miles</td>
<td>11%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>100 miles &lt; distance</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

- Increasing number of DCs reduces distance to customers and, we assume, increases level of service.
- Costs do not move in same direction, however
  - The fixed facility costs increase with each new DC opened
  - Outbound transportation (DC to Customer) decreases with each new DC opened
Sensitivity Analysis with Number of DCs

Total Cost versus Average Demand Distance

- Total Cost ($/week)
- Average Demand Distance (miles)

Graph showing the relationship between the number of DCs opened and total cost versus average demand distance.

 CTL.SC2x - Supply Chain Design
Lesson: Facility Location Models
Sensitivity Analysis with Number of DCs

Total Cost versus Percent of Demand within 25 miles

- **Total Cost** (in $/week)
- **Percent of Demand ≤ 25 miles**

Number of DCs Opened (forced)

- 0
- 1
- 2
- 3
- 4
- 5
- 6

Total Cost:
- $60,000
- $65,000
- $70,000
- $75,000
- $80,000
- $85,000
- $90,000
- $95,000

Percent of Demand within 25 miles of a DC:
Finding Optimal Solution

- Finding Optimal number and selection of DCs
  - Set $P_{\text{MIN}}=1$ and $P_{\text{MAX}}=5$
  - ... and re-solve for lowest cost.

<table>
<thead>
<tr>
<th>Total Cost</th>
<th>$66,285</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of DCs opened</td>
<td>3</td>
</tr>
<tr>
<td>Average Customer Distance (miles)</td>
<td>38.58</td>
</tr>
<tr>
<td>Average Demand Distance (miles)</td>
<td>20.78</td>
</tr>
<tr>
<td>Maximum Customer Distance (miles)</td>
<td>69 (CO)</td>
</tr>
</tbody>
</table>

Percent Demand within distance bracket:

- distance ≤ 25 miles | 55%
- 25 miles < distance ≤ 50 miles | 23%
- 50 miles < distance ≤ 75 miles | 22%
- 75 miles < distance ≤ 100 miles | 0%
- distance > 100 miles | 0%
Enforcing LOS Minimums
### Adding LOS Constraints

Minimize $z = \sum_i \sum_j c_{ij} x_{ij} + \sum_i f_i Y_i$

subject to:

- $\sum_j x_{ij} \leq S_i \quad \forall i \in S$
- $\sum_i x_{ij} \geq D_j \quad \forall j \in D$
- $x_{ij} - M_i Y_i \leq 0 \quad \forall ij$
- $\sum_i Y_i \geq P_{MIN}$
- $\sum_i Y_i \leq P_{MAX}$
- $\sum_j \left( \frac{d_{ij} x_{ij}}{\sum_j D_j} \right) \leq MaxAvgDist$
- $\sum_j \left( \frac{a_{ij} x_{ij}}{\sum_j D_j} \right) \geq MinPctIn50$
- $x_{ij} \geq 0 \quad \forall ij$
- $Y_i = \{0,1\} \quad \forall i$

**Indices**
- Distribution Centers $i$
- Customers $j$

**Decision Variables**
- $x_{ij} = \text{Flow on arc from DC } i \text{ to Customer } j (\text{units}) \quad \forall i, j$
- $Y_i = 1 \text{ if DC } i \text{ is opened}; =0 \text{ otherwise} \quad \forall i \in S$

**Input Data**
- $S_i = \text{Available supply at DC } i (\text{units}) \quad \forall i \in S$
- $D_j = \text{Demand by Customer } j (\text{units}) \quad \forall j \in D$
- $c_{ij} = \text{Cost to serve Customer } j \text{ from DC } i (\$/\text{unit}) \quad \forall i, j$
- $f_i = \text{Fixed cost for opening DC } i \text{ ($)} \quad \forall i \in S$
- $P_{MIN} = \text{Minimum number of DCs required to open}$
- $P_{MAX} = \text{Maximum number of DCs allowed to open}$
- $M = \text{A really big number, but not too big!}$
- $d_{ij} = \text{Distance to Customer } j \text{ from DC } i \text{ (miles)} \quad \forall i, j$
- $a_{ij} = 1 \text{ if Customer } j \text{ to DC } i \leq 50 \text{ miles}, =0 \text{ otherwise} \quad \forall i, j$

$MaxAvgDist = \text{Max allowable average distance DCs to Customers}$

$MinPctIn50 = \text{Min allowable demand within 50 miles of a DC}$

**d_{ij}**

<table>
<thead>
<tr>
<th></th>
<th>BO</th>
<th>BR</th>
<th>CO</th>
<th>HA</th>
<th>MN</th>
<th>NA</th>
<th>NH</th>
<th>NL</th>
<th>PO</th>
<th>PR</th>
<th>SP</th>
<th>WO</th>
</tr>
</thead>
<tbody>
<tr>
<td>BO</td>
<td>0</td>
<td>93</td>
<td>69</td>
<td>98</td>
<td>55</td>
<td>37</td>
<td>128</td>
<td>95</td>
<td>62</td>
<td>42</td>
<td>82</td>
<td>34</td>
</tr>
<tr>
<td>NA</td>
<td>37</td>
<td>65</td>
<td>33</td>
<td>103</td>
<td>20</td>
<td>0</td>
<td>137</td>
<td>113</td>
<td>48</td>
<td>72</td>
<td>79</td>
<td>41</td>
</tr>
<tr>
<td>PR</td>
<td>42</td>
<td>106</td>
<td>105</td>
<td>73</td>
<td>92</td>
<td>72</td>
<td>94</td>
<td>57</td>
<td>104</td>
<td>0</td>
<td>68</td>
<td>38</td>
</tr>
<tr>
<td>SP</td>
<td>82</td>
<td>59</td>
<td>101</td>
<td>27</td>
<td>93</td>
<td>79</td>
<td>63</td>
<td>57</td>
<td>127</td>
<td>68</td>
<td>0</td>
<td>47</td>
</tr>
<tr>
<td>WO</td>
<td>34</td>
<td>68</td>
<td>72</td>
<td>66</td>
<td>60</td>
<td>41</td>
<td>98</td>
<td>71</td>
<td>85</td>
<td>38</td>
<td>47</td>
<td>0</td>
</tr>
</tbody>
</table>

**a_{ij}**

<table>
<thead>
<tr>
<th></th>
<th>BO</th>
<th>BR</th>
<th>CO</th>
<th>HA</th>
<th>MN</th>
<th>NA</th>
<th>NH</th>
<th>NL</th>
<th>PO</th>
<th>PR</th>
<th>SP</th>
<th>WO</th>
</tr>
</thead>
<tbody>
<tr>
<td>BO</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>NA</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PR</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>SP</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>WO</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Adding LOS Constraints (NERD3)

| A | B | C   | D   | E   | F   | G   | H   | I   | J   | K   | L   | M   |
|---|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1 |   | NERD3 Facility Location | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 |   | z = $66,781 |
| 3 |   |              |   |   |   |   |   |   | d_{BO-BR}/\sum D_j = 93/1746 = 0.05326 |
| 4 |   | BO | NA | PR | SP | WD | BO | BO-BR | BO-CO | BO-HA | BO-MN | BO-NA |
| 95 |   |     |    |    |    |   |   |   |   |   |   |   |
| 96 | LOS Constraints | Min Avg Distance | MaxPct In 50 |
| 97 |   |   | - | 0.05 | 0.04 | 0.06 | 0.03 | 0.02 | 0.00 | - | - | - | 0.00 |
| 98 | [cell I97] = \frac{d_{BO-BR}}{\sum D_j} = \frac{93}{1746} = 0.05326 |

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>NERD3 Facility Location</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>z = $66,781</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>BO-BD</td>
<td>BO-BE</td>
<td>BO-BF</td>
<td>BO-BG</td>
<td>BO-BH</td>
<td>BO-BI</td>
<td>BO-BJ</td>
<td>BO-BK</td>
<td>BO-BL</td>
<td>BO-BM</td>
<td>BO-CN</td>
</tr>
<tr>
<td>95</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>96</td>
<td>LOS Constraints</td>
<td>Min Avg Distance</td>
<td>MaxPct In 50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>97</td>
<td></td>
<td></td>
<td>0.02</td>
<td>0.04</td>
<td>0.04</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
<td>0.06</td>
<td>0.04</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>98</td>
<td>[cell BP97] = \text{SUMPRODUCT}($C$5:$BO$5,C97:BO97)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[cell BP98] = \text{SUMPRODUCT}($C$5:$BO$5,C98:BO98)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[cell H98] = \frac{a_{BO-BO}}{\sum D_j} = \frac{1}{1746} = 0.0005727</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

[cell I97] = \frac{d_{BO-BR}}{\sum D_j} = \frac{93}{1746} = 0.05326

[cell H98] = \frac{a_{BO-BO}}{\sum D_j} = \frac{1}{1746} = 0.0005727

[cell BP97] = \text{SUMPRODUCT}($C$5:$BO$5,C97:BO97)
[cell BP98] = \text{SUMPRODUCT}($C$5:$BO$5,C98:BO98)
Solving NERD3

The new LOS constraints for:
(1) Maximum Average Distance and
(2) Minimum Percent Within 50 miles

Solution
4 DC Locations: SP, PR, NA, & BO
Average customer distance = 15 miles
Percent of customers ≤ 50 miles = 90%
Total Cost: $66,781 per week

Insights
- LOS constraints can drive the solution
- We can estimate the cost of meeting a specific LOS desire
- LOS can be specified in MANY different ways with different metrics
Key Take Aways
Key Take Aways

• Facility Location Problems
  ■ Continuous Selection
    ♦ Center of Gravity Model (volume x distance\(^2\))
    ♦ Weber Model (minimizes volume x distance)
  ■ Discrete Candidate Selection
    ♦ Network Location Models solved as a MILPs
    ♦ Trade offs between costs . . .
      ■ Variable outbound transportation costs
      ■ Fixed DC costs
    ♦ . . . subject to LOS and other constraints
    ♦ Use as a decision support tool
      ■ Informs the decision making process
      ■ Always conduct sensitivity analysis

\[
\begin{align*}
\text{Min} \quad z &= \sum_{k \in K} w_k d_k (x, y) \\
\text{Min} \quad z &= \sum_i \sum_j c_{ij} x_{ij} + \sum_i f_i Y_i \\
\text{s.t.} \quad \sum_j x_{ij} &\leq S_i \quad \forall i \in S \\
\sum_i x_{ij} &\geq D_j \quad \forall j \in D \\
x_{ij} - M_{ij} Y_{ij} &\leq 0 \quad \forall ij \\
\sum_i Y_i &\geq P_{MIN} \\
\sum_i Y_i &\leq P_{MAX} \\
\sum_{ij} \left( \frac{d_{ij} x_{ij}}{\sum_j D_j} \right) &\leq \text{MaxAvgDist} \\
\sum_{ij} \left( \frac{a_{ij} x_{ij}}{\sum_j D_j} \right) &\geq \text{MinPctIn50} \\
x_{ij} &\geq 0 \quad \forall ij \\
Y_i &= \{0, 1\} \quad \forall i
\end{align*}
\]
Key Take Aways

• Optimization models are decision support tools
  ■ They suggest solutions – they shouldn’t dictate
  ■ They are deterministic, assume perfect data, and do ANYTHING to save $1
  ■ They will NEVER include complete and perfect information

• Models can be used for sensitivity analysis to quantify the impact of:
  ■ Adding/removing/changing constraints (e.g. forcing DCs to open/close, changing DC capacity)
  ■ Changing input values (e.g., distances, demand, etc.)
  ■ Modifying LOS requirements (e.g., maximum distance from DC, average distance, minimum within certain distance, etc.)
  ■ Changing objective function values (e.g., costs)
Key Take Aways

- A sampling of the things we did NOT consider (yet):
  - Capacity on arcs or nodes
  - Manufacturing locations (3 tiered networks)
  - Inbound transportation costs
  - Variable handling costs at facilities
  - Fixed transportation costs
  - Inventory costs (cycle, safety, pipeline)
  - Multiple products, categories, or families
  - Multiple time periods
  - Formal analysis of optimization models
Questions, Comments, Suggestions?
Use the Discussion!

“Dexter & Wilson – hoping to optimize their location next to the food bowl”
Yankee Golden Retriever Rescued Dogs (www.ygrr.org)