

Module 5 Practice problem and Homework answers

Practice problem

What is the mean for the before period?

Answer: 5.3

$$\bar{x} = \frac{74.1}{14} = 5.3$$

What is the mean for the after period?

Answer: 6.8

$$\bar{x} = \frac{95.3}{14} = 6.8$$

What is the sum of squared deviations for the before group?

Answer: 28.23

What is the sum of squared deviations for the after group?

Answer: 31.35

x_1 (before)	\bar{x}_1	$x_1 - \bar{x}_1$	$(x_1 - \bar{x}_1)^2$		x_2 (after)	\bar{x}_2	$x_2 - \bar{x}_2$	$(x_2 - \bar{x}_2)^2$
4.2	5.3	-1.1	1.21		8	6.8	1.2	1.44
7.7	5.3	2.4	5.76		6.2	6.8	-0.6	0.36
3.3	5.3	-2.0	4.00		4.1	6.8	-2.7	7.29
4.7	5.3	-0.6	0.36		7.3	6.8	0.5	0.25
4.9	5.3	-0.4	0.16		5.4	6.8	-1.4	1.96
5.1	5.3	-0.2	0.04		6.1	6.8	-0.7	0.49
6.8	5.3	1.5	2.25		7.5	6.8	0.7	0.49
4.7	5.3	-0.6	0.36		7.8	6.8	1.0	1.00
3.3	5.3	-2.0	4.00		9.5	6.8	2.7	7.29
7	5.3	1.7	2.89		4.5	6.8	-2.3	5.29
5.1	5.3	-0.2	0.04		8.2	6.8	1.4	1.96
5.9	5.3	0.6	0.36		5.9	6.8	-0.9	0.81
7.5	5.3	2.2	4.84		6.4	6.8	-0.4	0.16
3.9	5.3	-1.4	1.96		8.4	6.8	1.6	2.56
			$\Sigma = 28.23$					$\Sigma = 31.35$

Practice problem, continued

What is the standard deviation of the before group?

Answer: 1.42

$$s_1 = \sqrt{\frac{\sum(x_1 - \bar{x}_1)^2}{N}} = \sqrt{\frac{28.23}{14}} = 1.42$$

What is the standard deviation of the after group?

Answer: 1.50

$$s_2 = \sqrt{\frac{\sum(x_2 - \bar{x}_2)^2}{N}} = \sqrt{\frac{31.35}{14}} = 1.50$$

What is the value of the standard error of the difference of means? (What is the result of the sigma formula?)

Answer: 0.562

$$\begin{aligned}\sigma_{\bar{x}_1 - \bar{x}_2} &= \sqrt{\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}} \sqrt{\frac{N_1 + N_2}{N_1 N_2}} \\ \sigma_{\bar{x}_1 - \bar{x}_2} &= \sqrt{\frac{14 * 1.42^2 + 14 * 1.50^2}{14 + 14 - 2}} \sqrt{\frac{14 + 14}{14 * 14}} \\ &= \sqrt{\frac{14 * 2.02 + 14 * 2.25}{26}} \sqrt{\frac{28}{196}} = \sqrt{\frac{28.28 + 31.50}{26}} \sqrt{0.14} \\ &= \sqrt{2.30} \sqrt{0.14} = 1.52 * 0.37 = 0.562\end{aligned}$$

What is the value of t?

Answer: -2.67

$$\begin{aligned}t &= \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}} \\ t &= \frac{5.3 - 6.8}{0.562} = \frac{-1.5}{0.562} = -2.67\end{aligned}$$

Practice problem, continued

What is the df for this problem?

Answer: 26

$$df = N_1 + N_2 - 2 = 14 + 14 - 2 = 26$$

What is the critical t value?

Answer: 1.706

For a one-tailed test at .05 significance level and 26 degrees of freedom, the degrees of freedom value is 1.706.

Is the difference between groups significant?

Answer: yes

Because the calculated value (-2.67) has a higher magnitude than the critical value (1.706), we can reject the null hypothesis and conclude that there is a significant difference between groups.

Was the manager correct that the discount increased revenue, or was her boss correct that there was no significant difference in sales before and after the discount? Who was right?

Answer: the manager

Because the difference between groups is significant, and because the mean sales for the before period was greater than the mean for the after period, the manager was right.

Homework problems

page 1

Is the research hypothesis one-tailed or two-tailed?

Answer: two-tailed

The set-up to this problem says that the research hypothesis is that the number of emails varies significantly by project, which predicts that there is a significant difference between projects, but not which project will have the greater number of emails. Because there is no specific direction indicated in this hypothesis, it is two-tailed.

Calculate the mean for Project A.

Answer: 19.7

$$\bar{x} = \frac{197}{10} = 19.7$$

Homework page 1, continued

Calculate the mean for Project B.

Answer: 24.9

$$\bar{x} = \frac{249}{10} = 24.9$$

Calculate the standard deviation for Project A.

Answer: 3.13

Project A	mean	deviation	squared deviation
17	19.7	-2.7	7.29
24	19.7	4.3	18.49
23	19.7	3.3	10.89
18	19.7	-1.7	2.89
15	19.7	-4.7	22.09
21	19.7	1.3	1.69
17	19.7	-2.7	7.29
21	19.7	1.3	1.69
17	19.7	-2.7	7.29
24	19.7	4.3	18.49
			$\Sigma = 98.10$

$$s_1 = \sqrt{\frac{\Sigma(x_1 - \bar{x}_1)^2}{N}} = \sqrt{\frac{98.10}{10}} = 3.13$$

Homework page 1, continued

Calculate the standard deviation for Project B.

Answer: 7.03

Project B	mean	deviation	squared deviation
28	24.9	3.1	9.61
30	24.9	5.1	26.01
32	24.9	7.1	50.41
13	24.9	-11.9	141.61
32	24.9	7.1	50.41
28	24.9	3.1	9.61
20	24.9	-4.9	24.01
19	24.9	-5.9	34.81
32	24.9	7.1	50.41
15	24.9	-9.9	98.01
			$\Sigma = 494.90$

$$s_2 = \sqrt{\frac{\sum(x_2 - \bar{x}_2)^2}{N}} = \sqrt{\frac{494.90}{10}} = 7.03$$

Calculate t .

Answer: -2.02

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}}, \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}} \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$

$$\begin{aligned} \sigma_{\bar{x}_1 - \bar{x}_2} &= \sqrt{\frac{10 * 3.13^2 + 10 * 7.03^2}{10 + 10 - 2}} \sqrt{\frac{10 + 10}{10 * 10}} \\ &= \sqrt{\frac{10 * 9.80 + 10 * 49.42}{18}} \sqrt{\frac{20}{100}} = \sqrt{\frac{98.0 + 494.2}{18}} \sqrt{0.2} = \sqrt{32.9} \sqrt{.2} \\ &= 5.74 * 0.45 = 2.58 \end{aligned}$$

$$t = \frac{19.7 - 24.9}{2.58} = \frac{-5.2}{2.58} = -2.02$$

Homework page 1, continued

What is the df?

Answer: 18

$$df = N_1 + N_2 - 2 = 10 + 10 - 2 = 18$$

What is the critical t value?

Answer: 2.101

For a two-tailed test at .05 significance level and 18 degrees of freedom, the degrees of freedom value is 2.101.

Is the difference between groups significant?

Answer: no

Because the calculated value (-2.02) has a lower magnitude than the critical value (2.101), we fail to reject the null hypothesis and conclude that there is not a significantly different number of emails between projects.

page 2

Is the research hypothesis one-tailed or two-tailed?

Answer: one-tailed

The set-up to this problem says that the research hypothesis is that one salon has had a significantly higher number of recent new clients than the other one has. Because this hypothesis predicts a specific direction, it is one-tailed.

Calculate the t value.

Answer: 1.69

$$\begin{aligned} t = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}}, \sigma_{\bar{x}_1 - \bar{x}_2} &= \sqrt{\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}} \sqrt{\frac{N_1 + N_2}{N_1 N_2}} \\ \sigma_{\bar{x}_1 - \bar{x}_2} &= \sqrt{\frac{26 * 2.86^2 + 26 * 2.95^2}{26 + 26 - 2}} \sqrt{\frac{26 + 26}{26 * 26}} \\ &= \sqrt{\frac{26 * 8.18 + 26 * 8.70}{50}} \sqrt{\frac{52}{676}} = \sqrt{\frac{212.68 + 226.20}{50}} \sqrt{0.08} = \sqrt{8.78} \sqrt{0.08} \\ &= 2.96 * 0.28 = 0.83 \end{aligned}$$

Homework page 2, continued

$$t = \frac{9.2 - 7.8}{0.83} = \frac{1.4}{0.83} = 1.69$$

What is the df?

Answer: 50

$$df = N_1 + N_2 - 2 = 26 + 26 - 2 = 50$$

What is the critical t value?

Answer: 1.671

For a one-tailed test at .05 significance level and 50 degrees of freedom, the degrees of freedom value is 1.671. (Note: when the specific df value you're working with isn't in the t table, round up to the next value. In this case, we've used df=60.)

Is the difference between groups significant?

Answer: yes

Because the calculated value (1.69) has a higher magnitude than the critical value (1.671), we can reject the null hypothesis and conclude that the downtown salon did have a significantly greater number of recent new clients than the suburb salon did.

Homework page 3

Is the research hypothesis one-tailed or two-tailed?

Answer: one-tailed

The research hypothesis suggests that the amount of recyclable material that is thrown in the trash will be significantly lower after the campaign than before it. Because one period is predicted to be significantly higher than the other, this is a one-tailed hypothesis.

Homework page 3, continued

Calculate the t value.

Answer: 4.59

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sigma_{\bar{x}_1 - \bar{x}_2}}, \sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{N_1 s_1^2 + N_2 s_2^2}{N_1 + N_2 - 2}} \sqrt{\frac{N_1 + N_2}{N_1 N_2}}$$
$$\sigma_{\bar{x}_1 - \bar{x}_2} = \sqrt{\frac{31 * 1.43^2 + 30 * 1.41^2}{31 + 30 - 2}} \sqrt{\frac{31 + 30}{31 * 30}}$$
$$= \sqrt{\frac{31 * 2.04 + 30 * 1.99}{59}} \sqrt{\frac{61}{930}} = \sqrt{\frac{63.24 + 59.70}{59}} \sqrt{0.07} = \sqrt{2.08} \sqrt{.07}$$
$$= 1.44 * 0.26 = 0.37$$
$$t = \frac{19.6 - 17.9}{0.37} = \frac{1.7}{0.37} = 4.59$$

What is the df ?

Answer: 59

$$df = N_1 + N_2 - 2 = 30 + 31 - 2 = 59$$

What is the critical t value?

Answer: 1.671

For a one-tailed test at .05 significance level and 59 degrees of freedom, the degrees of freedom value is 1.671.

Is the difference between groups significant?

Answer: yes

Because the calculated value (4.59) has a higher magnitude than the critical value (1.671), we can reject the null hypothesis and conclude that there was significantly less recyclable material thrown in the trash after the campaign than before it.