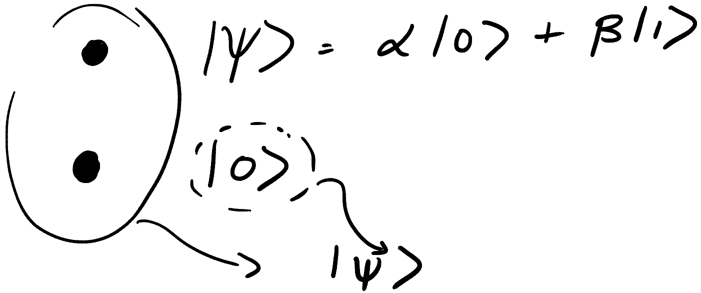
Quantum Mechanics & Quantum Computation

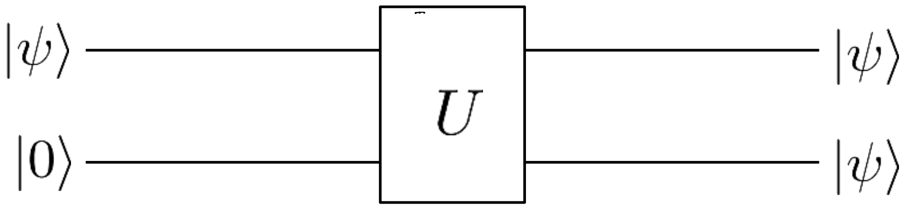
Umesh V. Vazirani
University of California, Berkeley

Lecture 6: Quantum Teleportation

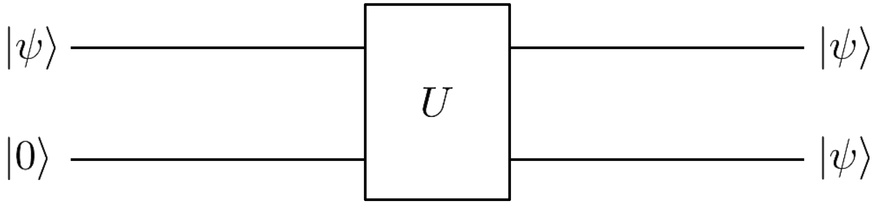
No Cloning Theorem

No Cloning Theorem





- Construct a quantum circuit for copying a quantum bit.



$$(\alpha|0\rangle + \beta|1\rangle) \otimes |0\rangle \xrightarrow{U} (\alpha|0\rangle + \beta|1\rangle) \otimes (\alpha|0\rangle + \beta|1\rangle) \quad \forall \alpha, \beta \in \mathbb{C}.$$

$$= \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$$

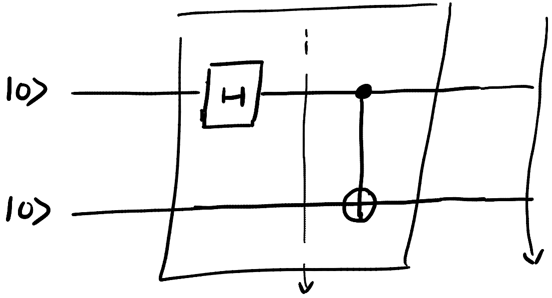
$$|00\rangle \xrightarrow{U} |00\rangle$$

$$|10\rangle \xrightarrow{U} |11\rangle$$

$$\alpha|00\rangle + \beta|10\rangle \xrightarrow{U} \alpha U|00\rangle + \beta U|10\rangle$$

$$= \alpha|00\rangle + \beta|11\rangle$$

$$\alpha = 1 \text{ or } \beta = 1. \quad \times$$



$$|00\rangle \xrightarrow{H} \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)|0\rangle$$

$$= \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle \xrightarrow{CNOT} \underline{\underline{\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle}} = |\Phi^+\rangle$$

$$|01\rangle \xrightarrow{H} \left(\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle\right)|1\rangle$$

$$= \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|11\rangle \xrightarrow{CNOT} \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle = |\Psi^+\rangle$$

$$|10\rangle \xrightarrow{H} \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)|0\rangle$$

$$= \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

$$\downarrow CNOT$$

$$\frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle = |\Phi^-\rangle$$

$$|11\rangle \xrightarrow{H} \left(\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle\right)|1\rangle$$

$$= \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

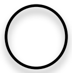
$$\downarrow CNOT$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

$|\Phi^\pm\rangle$ & $|\Psi^\pm\rangle$ Bell Basis States.

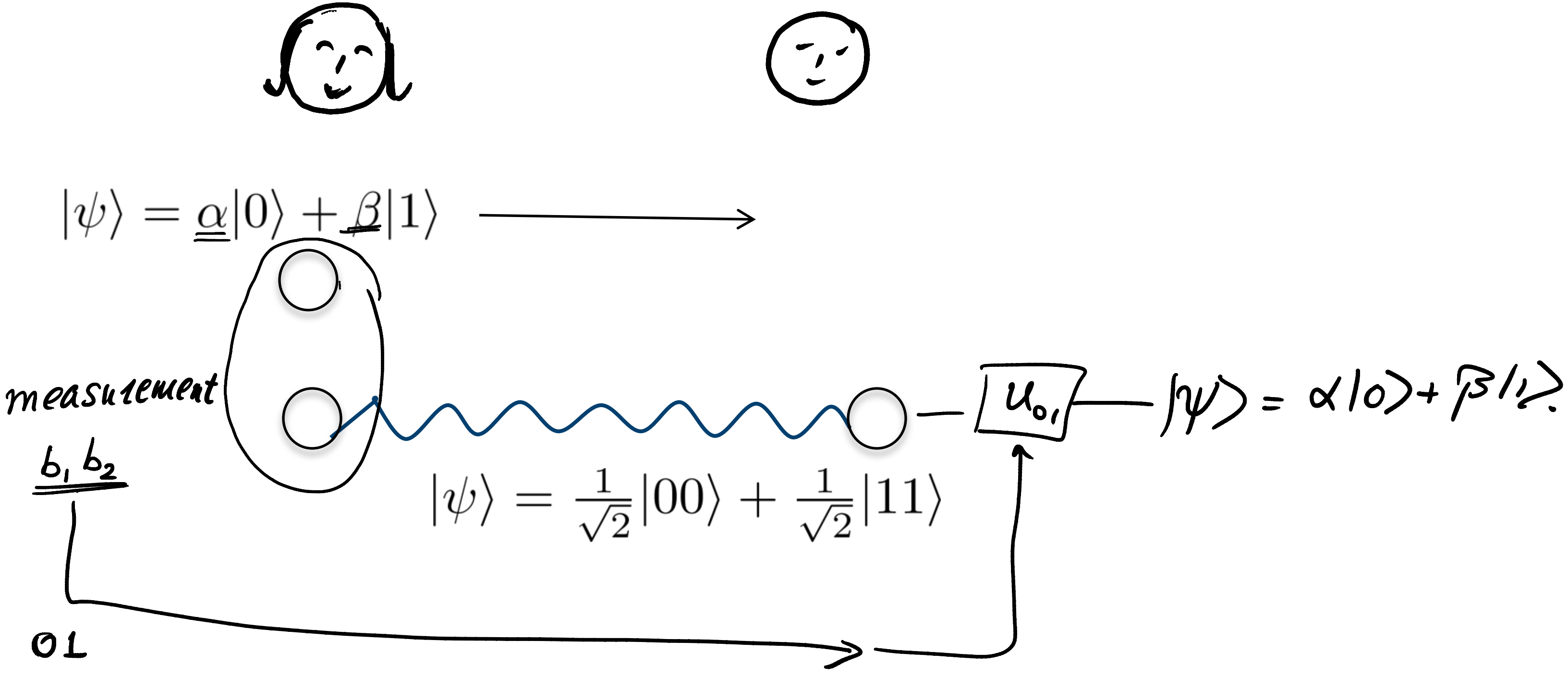
Quantum teleportation

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \longrightarrow$$



Quantum teleportation

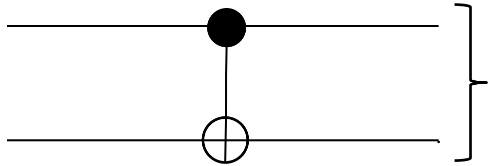
- It is impossible to clone quantum information, but it is possible to **teleport** a quantum state to another location.



Assume CNOT

Alice's qubit $\alpha|0\rangle + \beta|1\rangle$

Bob's qubit $|0\rangle$



$\alpha|00\rangle + \beta|11\rangle$

$+$ $\downarrow 0$ $-$ $\downarrow 1$

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

$\alpha|00\rangle + \beta|10\rangle$

Alice : Measure her qubit.

0	$\|00\rangle$	$\|0\rangle$ X
1	$\|11\rangle$	$\|1\rangle$

Measure in +/- basis :

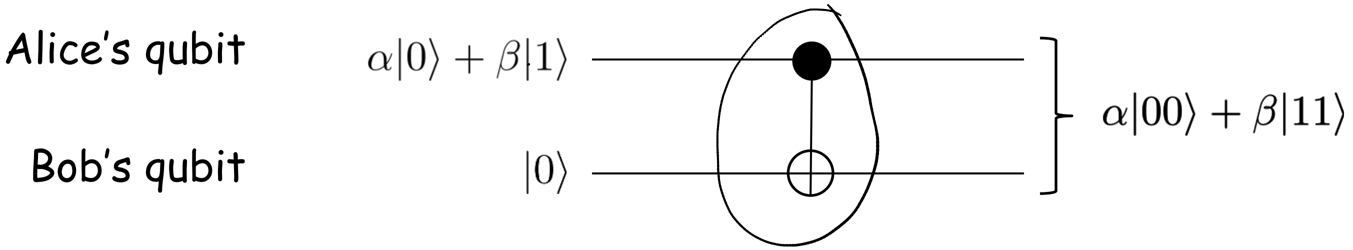
$$\alpha|00\rangle + \beta|11\rangle = \alpha\left(\tfrac{1}{\sqrt2}|+\rangle + \tfrac{1}{\sqrt2}|-\rangle\right)\otimes|0\rangle + \beta\left(\tfrac{1}{\sqrt2}|+\rangle - \tfrac{1}{\sqrt2}|-\rangle\right)|1\rangle$$

$$= \tfrac{1}{\sqrt2}|+\rangle\left[\alpha|0\rangle + \beta|1\rangle\right] + \tfrac{1}{\sqrt2}|-\rangle\left[\alpha|0\rangle - \beta|1\rangle\right]$$

$+$: New state $= |+\rangle\left[\alpha|0\rangle + \beta|1\rangle\right] = |\psi\rangle$

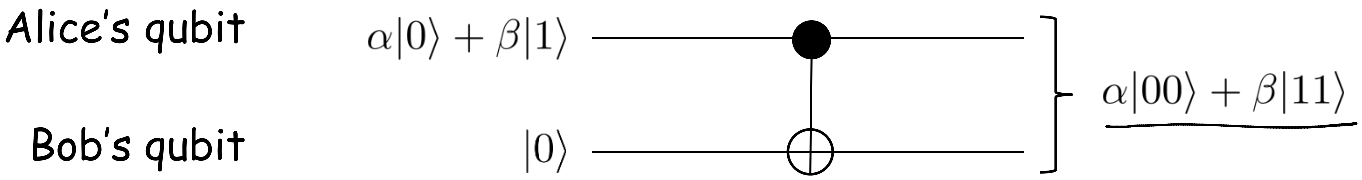
$-$: New State $= |-\rangle\left[\alpha|0\rangle - \beta|1\rangle\right]$ $Z\left[\alpha|0\rangle - \beta|1\rangle\right] = |\psi\rangle$.

Assume CNOT



Challenge: create the entangled state $\alpha|00\rangle + \beta|11\rangle$ without quantum communication between Alice and Bob!

Teleportation using CNOT



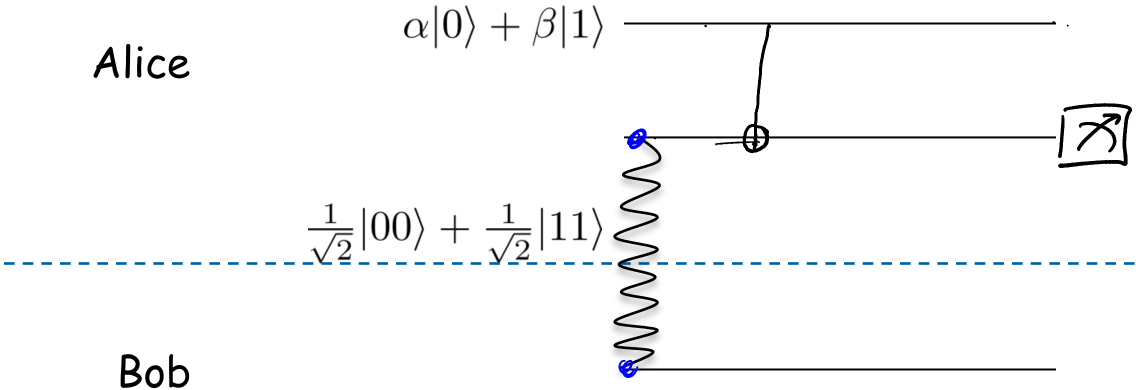
Alice measures in the sign basis:

$$
\begin{aligned}
+ &\longrightarrow |+\rangle \otimes (\alpha|0\rangle + \beta|1\rangle) \\
- &\longrightarrow |-\rangle \otimes (\alpha|0\rangle - \beta|1\rangle)
\end{aligned}
$$

If measurement result is -, Alice calls Bob and tells him to flip the phase.

But they are far apart…

- Suppose they share a Bell state.
- Can we use it to effectively apply CNOT remotely?



0 $\quad \alpha|00\rangle + \beta|11\rangle$

1 $\quad \alpha|01\rangle + \beta|10\rangle$

$\rightarrow \alpha|00\rangle + \beta|11\rangle$

$$(\alpha|0\rangle + \beta|1\rangle)\left(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle\right)$$

$$= \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|100\rangle + \frac{\beta}{\sqrt{2}}|111\rangle \xrightarrow{\text{CNOT}} \frac{\alpha}{\sqrt{2}}|000\rangle + \frac{\beta}{\sqrt{2}}|101\rangle + \frac{\alpha}{\sqrt{2}}|011\rangle + \frac{\beta}{\sqrt{2}}|110\rangle$$

Complete quantum teleportation protocol