# REVENUE MANAGEMENT

## An Introduction to Linear Optimization

15.071x – The Analytics Edge

# Airline Regulation (1938-1978)

- The Civil Aeronautics Board (CAB) set fares, routes, and schedules for all interstate air transport
- Most major airlines favored this system due to guaranteed profits
- Led to inefficiency and higher costs
  - Applications for new routes and fares often delayed or dismissed

# Airline Deregulation (1978)

- The administration of President Jimmy Carter passed the Airline Deregulation Act in 1978
- The Act encouraged
  - **More competition**: 52 new airlines between 1980 and 2000
  - **New air routes**: saved passengers an estimated $10.3 billion each year in travel time
  - **Lower fares**: ticket prices are 40% lower today than they were in 1978
- This led to **more passengers**
  - The number of air passengers increased from 207.5 million in 1974 to 721.1 million in 2010

# A Competitive Edge

- More competition led to heavy losses by air carriers
  - Need to lower fares while meeting operating costs
- 9 major carriers and more than 100 smaller airlines went bankrupt between 1978 and 2002
- How did airlines compete?

# Discount Fares

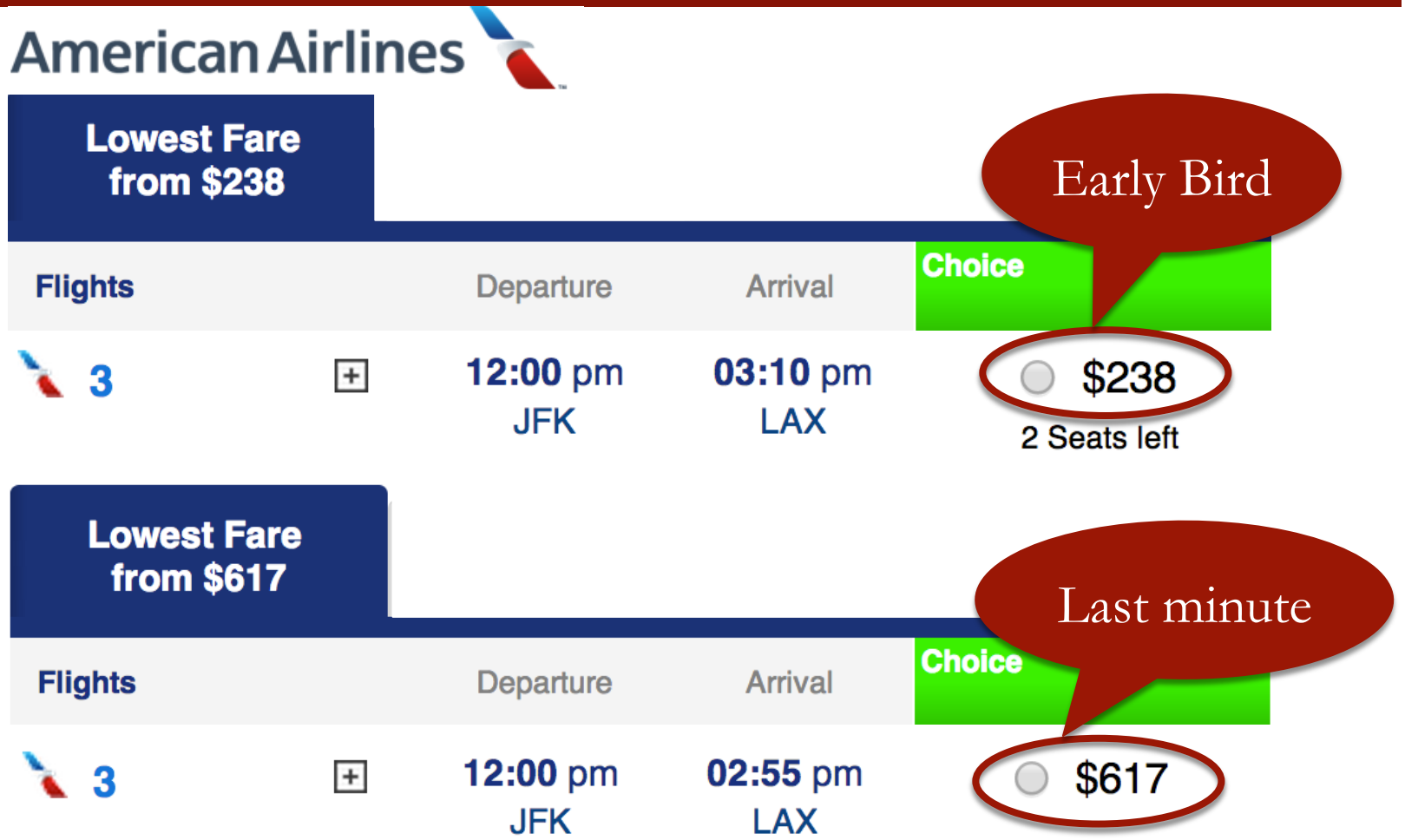
- On January 17, 1985 American Airlines (AA) launched its Ultimate Super Saver fares to compete with PeopleExpress
- Need to fill at least a minimum number of seats without selling every seat at discount prices
  - Sell enough seats to cover fixed operating costs
  - Sell remaining seats at higher rates to maximize revenues/profits

# How Many Seats to Sell on Discount?

- Passengers have different valuations
  - Business people value flexibility (last-minute/refundable)
  - People seeking getaways value good deals (early birds)
- Sell too many discounted seats
  - Not enough seats for high-paying passengers
- Sell too few discounted seats
  - Empty seats at takeoff implying lost revenue
- How should AA allocate its seats among customers to maximize its revenue?

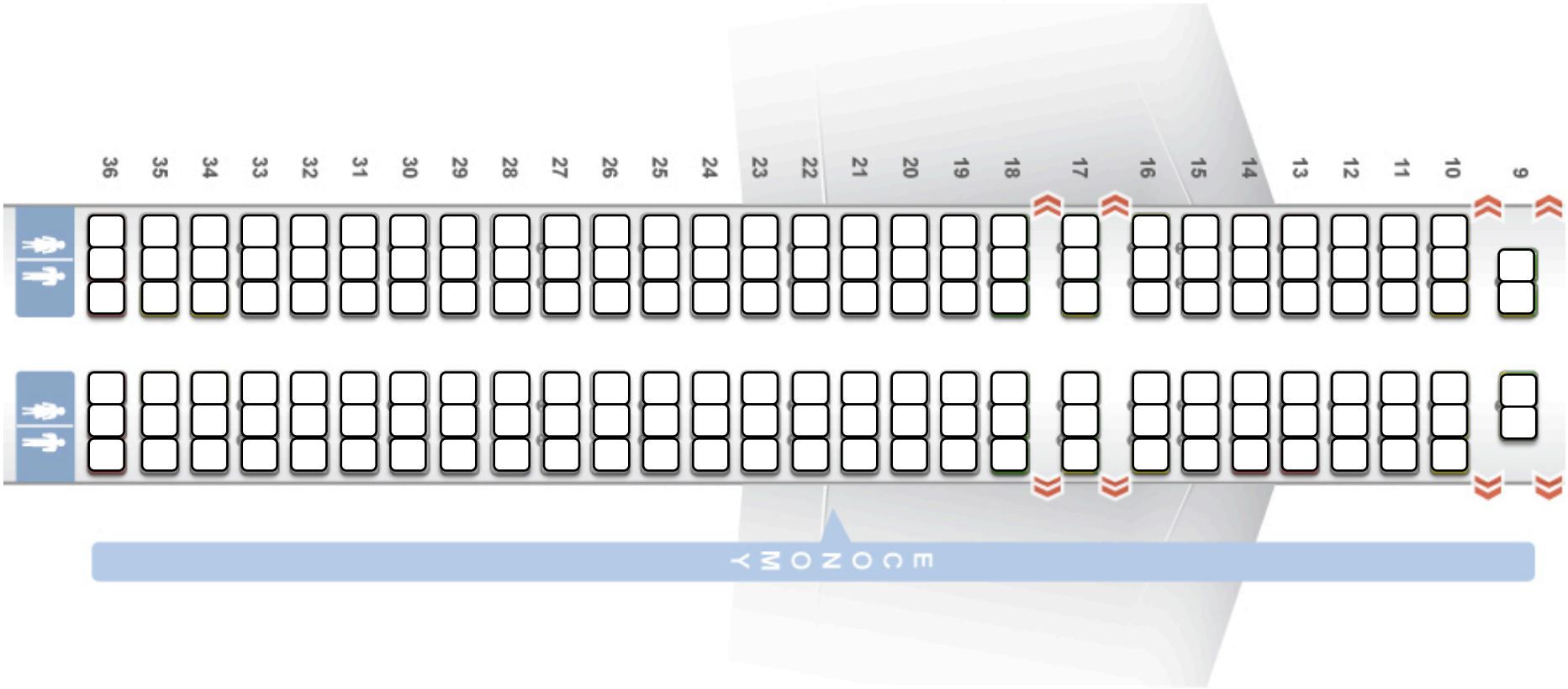
# Let's Start Simple

# Ticket Prices



# Boeing 757-200 Seat Map

- 166 Economy seats



# Demand Forecasting

- Demand for different prices can be forecasted using analytics tools, looking at historical data and incorporating models of human behavior
  - Time series methods
  - Linear regression
- Forecasts could be erroneous
  - Need to assess sensitivity to forecast errors
- We'll assume that demand has been forecasted

# Myopic Solution

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 50 | 50 |
| | Discount | 238 | 150 | 116 |

**Capacity 166**

- How many discount seats to sell to maximize revenue?

# Myopic Solution

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 100 | 100 |
| | Discount | 238 | 150 | 66 |

Capacity 166

- How many discount seats to sell to maximize revenue?

# Myopic Solution

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 200 | 166 |
| | Discount | 238 | 150 | 0 |

**Capacity 166**

- How many discount seats to sell to maximize revenue?
- This seems simple, but what if we had 100 different flights?
- In the next video, we'll see how to formulate this mathematically

# Single Route Example

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 100 | |
| | Discount | 238 | 150 | |

Capacity 166

- Problem: Find the optimal number of discounted seats and regular seats to sell to maximize revenue

- Let's formulate the problem mathematically

# Step 1. Decisions

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 100 | |
| | Discount | 238 | 150 | |

Capacity 166

- What are our decisions?
  - Number of regular seats to sell – $R$
  - Number of discount seats to sell – $D$

# Step 2. Objective

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 100 | |
| | Discount | 238 | 150 | |

Capacity 166

- What is our objective?
  - Maximizing total airline revenue
  - Revenue from each type of seat is equal to the number of that type of seat sold times the seat price

max 617*R + 238*D

# Step 3. Constraints

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 100 | |
| | Discount | 238 | 150 | |

Capacity 166

- AA cannot sell more seats than the aircraft capacity
  - Total number of seats sold cannot exceed capacity

    $R + D \leq 166$
- AA cannot sell more seats than the demand
  - Regular seats sold cannot exceed 100 $R \leq 100$
  - Discount seats sold cannot exceed 150 $D \leq 150$

# Step 4. Non-Negativity

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 100 | |
| | Discount | 238 | 150 | |

Capacity 166

- AA cannot sell a negative number of seats

$R \geq 0$ $D \geq 0$

# Problem Formulation

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 100 | |
| | Discount | 238 | 150 | |

Capacity 166

Maximize Total airline revenue

Subject to Seats sold cannot exceed capacity

Seats sold cannot exceed demand

Seats sold cannot be negative

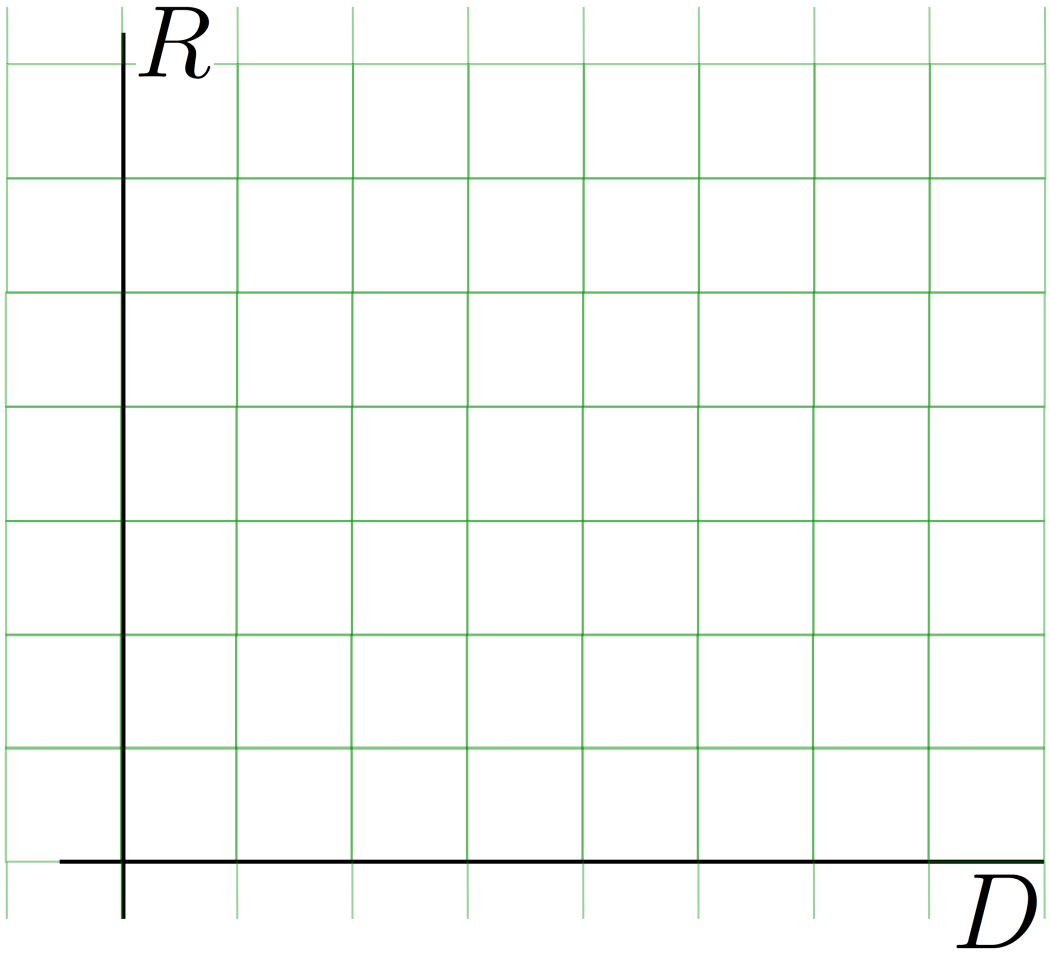
# Problem Formulation

| | | Price | Demand | Seats to Sell |
|---|---|---|---|---|
| JFK - LAX | Regular | 617 | 100 | |
| | Discount | 238 | 150 | |

Capacity 166

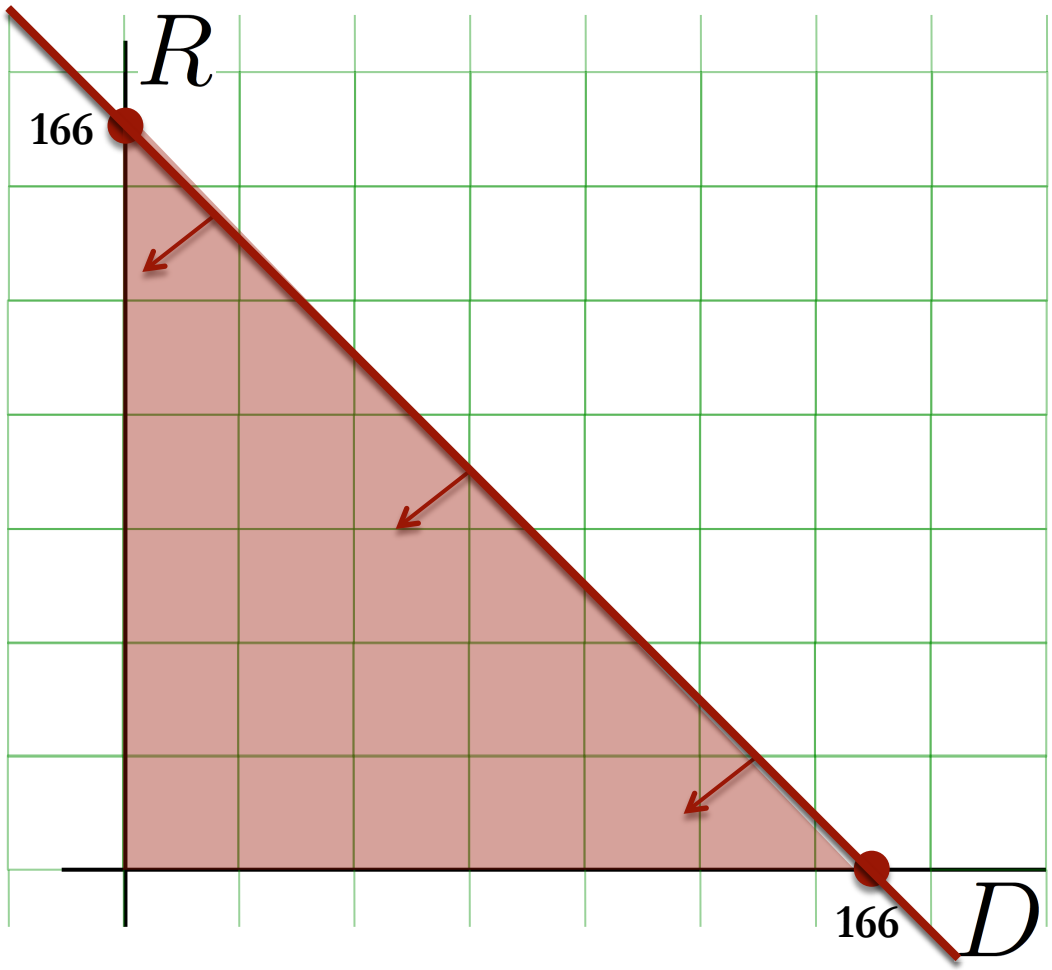
$$\begin{aligned} \text{Maximize} \quad & 617R + 238D \\ \text{Subject to} \quad & R + D \leq 166 \\ & R \leq 100, D \leq 150 \\ & R \geq 0, D \geq 0 \end{aligned}$$

# Visualizing the Problem



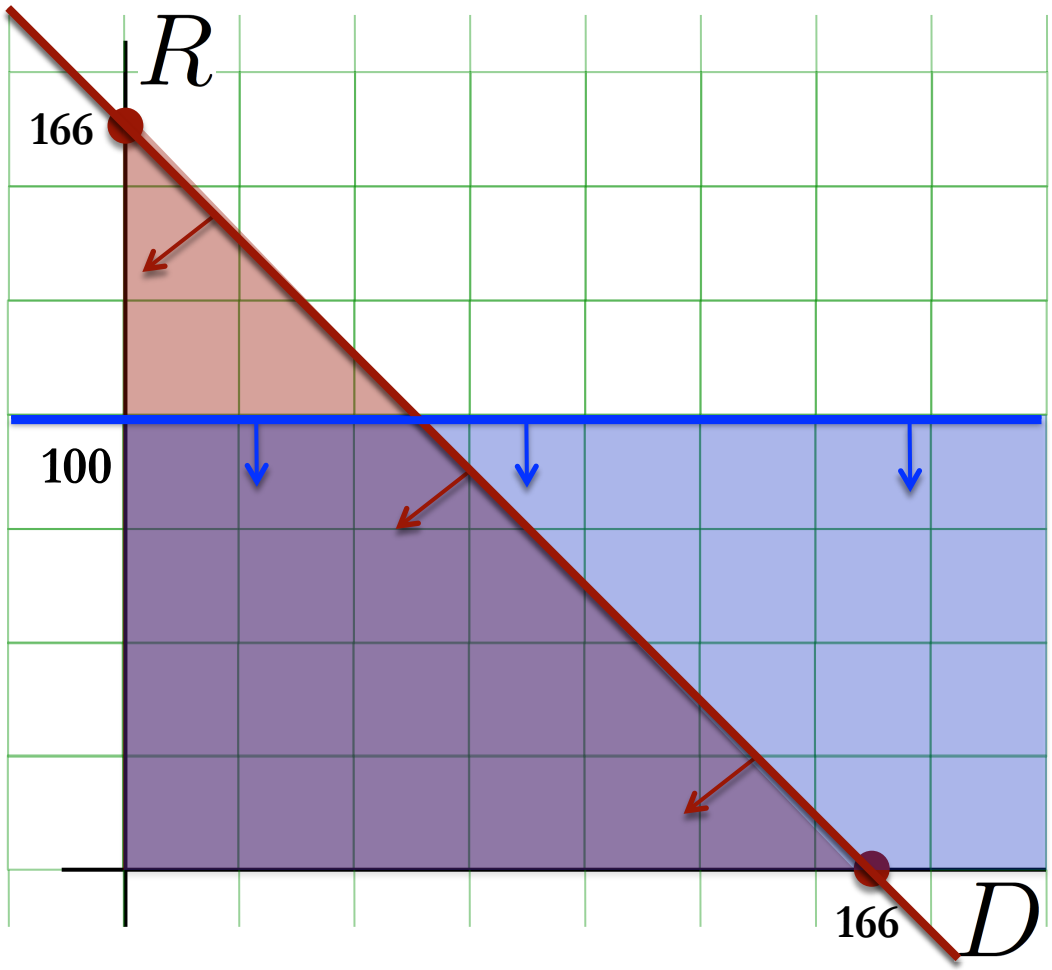

- 2D Representation
- Constraints
  - Non-negativity
    $R \geq 0, D \geq 0$

# Visualizing the Problem



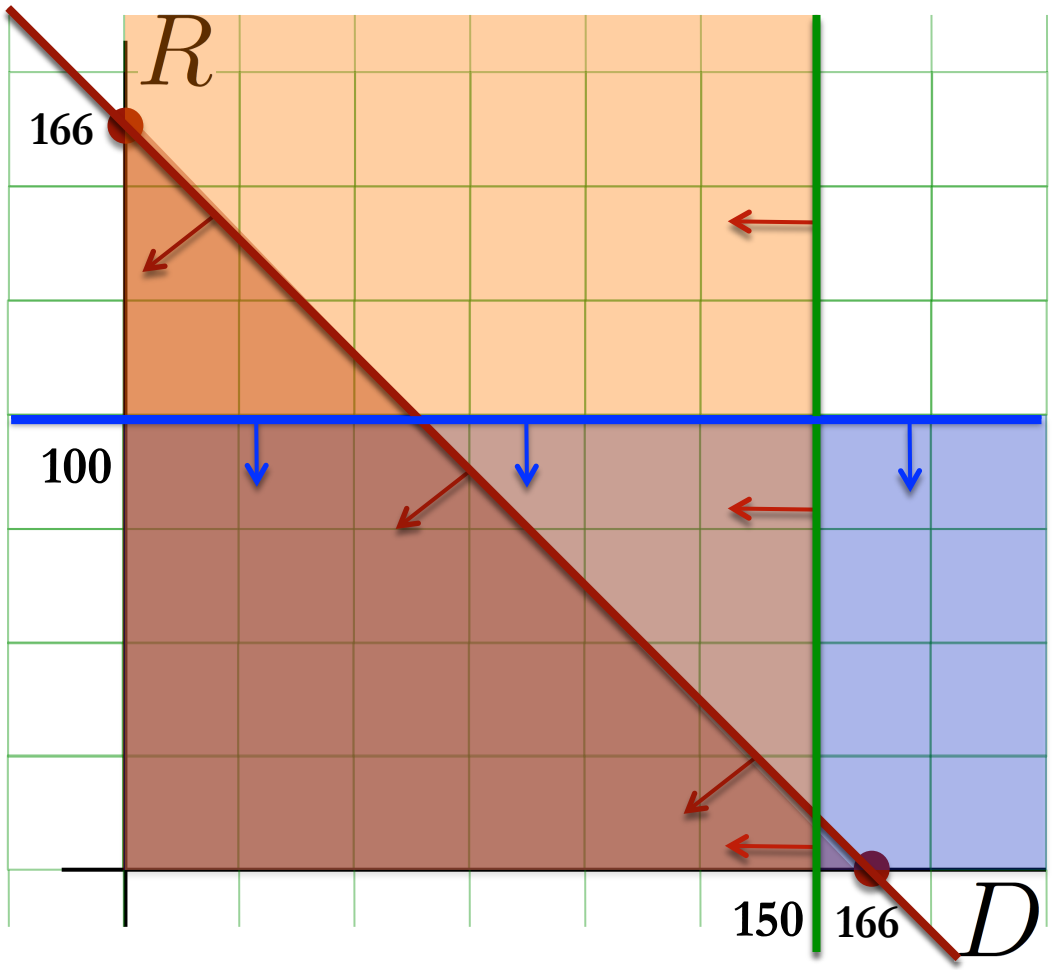

- 2D Representation
- Constraints
  - Non-negativity
    $R \geq 0, D \geq 0$
  - Capacity
    $R + D \leq 166$

# Visualizing the Problem



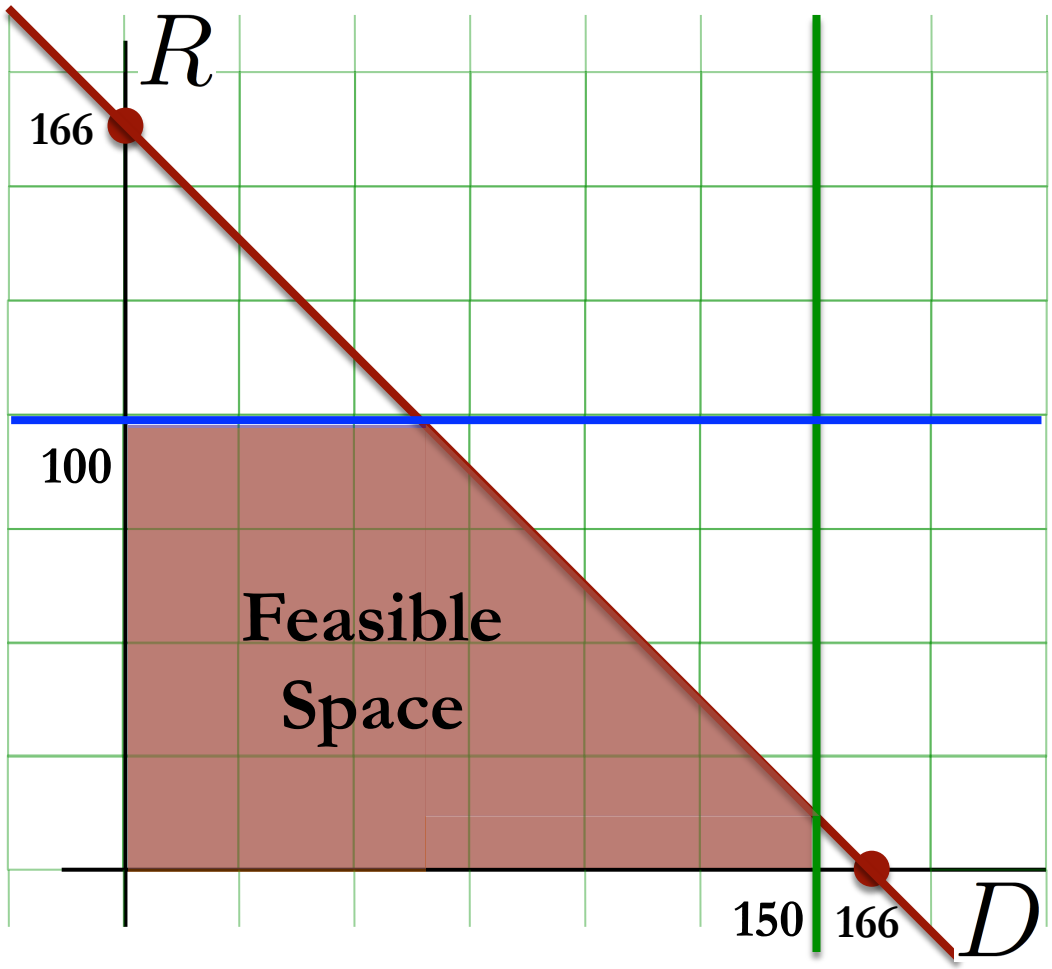

- 2D Representation
- Constraints
  - Non-negativity
    $R \geq 0, D \geq 0$
  - Capacity
    $R + D \leq 166$
  - Demand
    $R \leq 100, D \leq 150$

# Visualizing the Problem

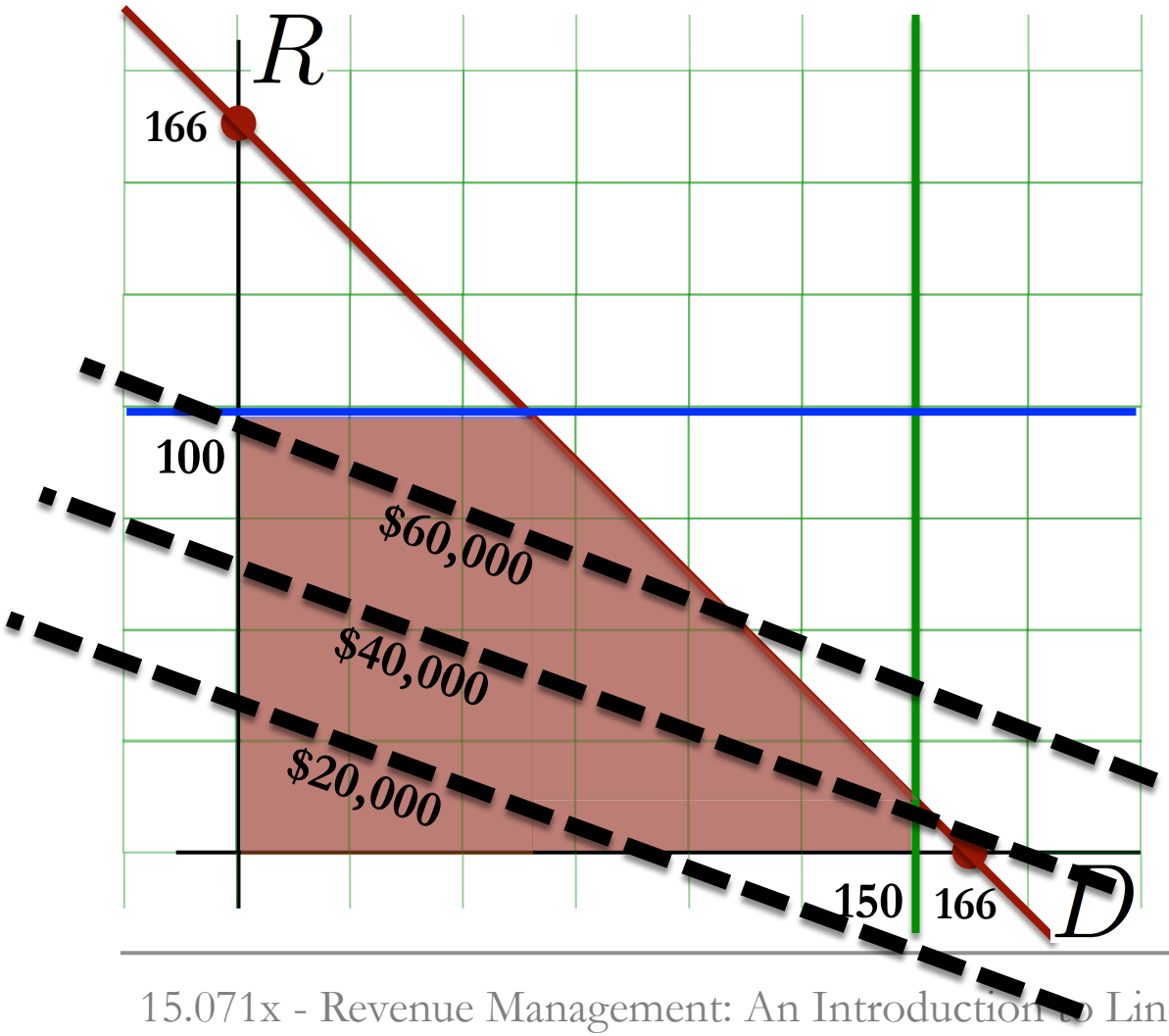



- 2D Representation
- Constraints
  - Non-negativity
    $R \geq 0, D \geq 0$
  - Capacity
    $R + D \leq 166$
  - Demand
    $R \leq 100, D \leq 150$

# Feasible Space

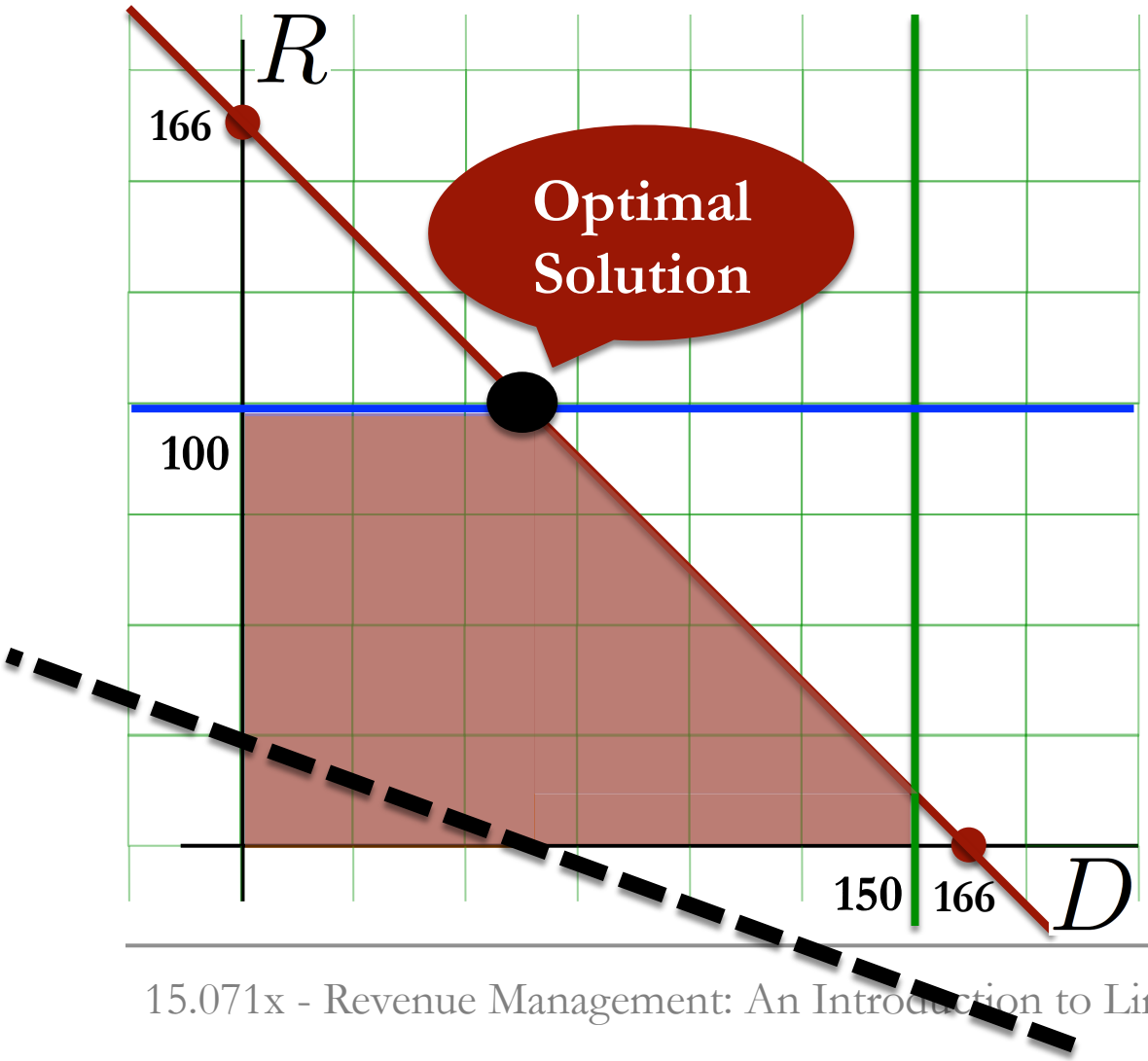


# Possible Solutions




- Revenue $617R + 238D$
- How many seats to sell of each type to achieve a revenue of
  - $20,000?
  - $40,000?
  - $60,000?

# Best Solution




- Revenue $617R + 238D$
- How many seats to sell of each type to achieve the highest revenue possible?

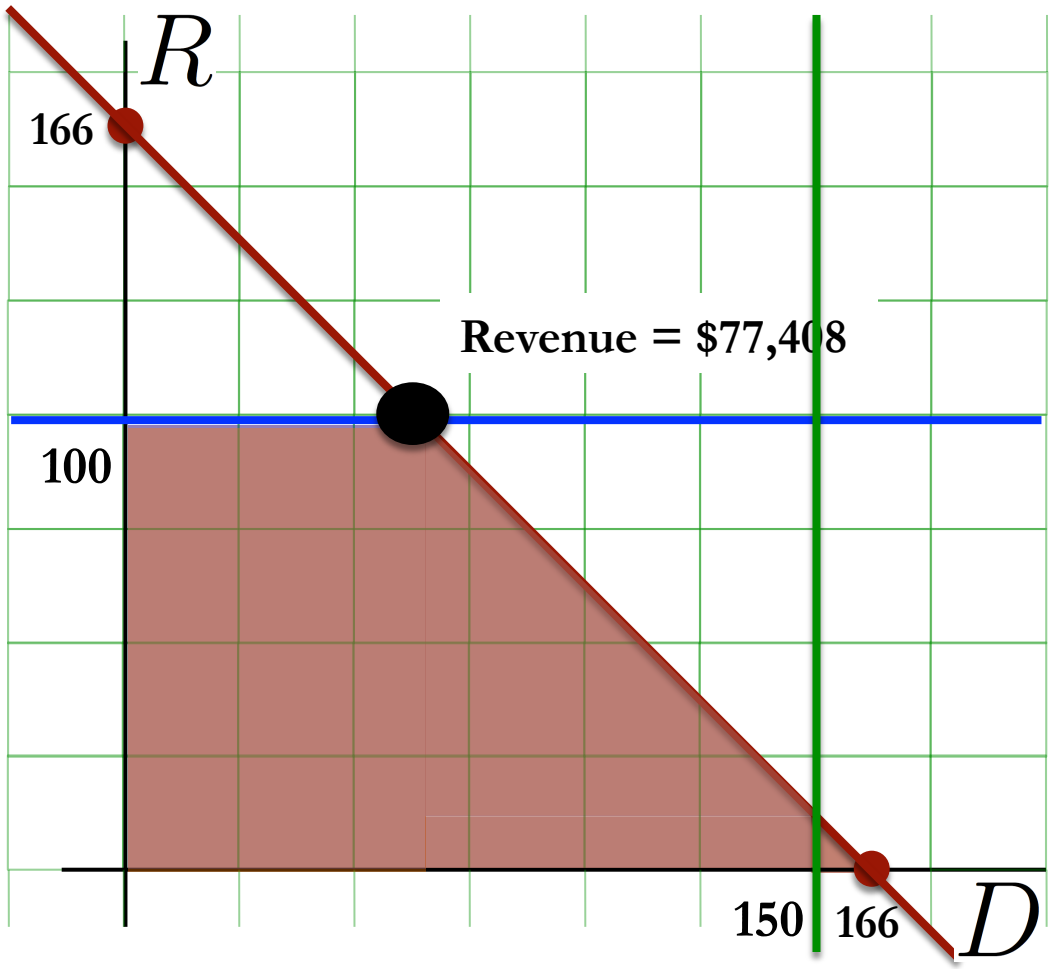
# Marketing Decisions

- Management is trying to figure out whether it would be beneficial to invest in marketing its fares

- AA forecasts that its marketing effort is likely to attract one more unit of demand per **$200 spent**

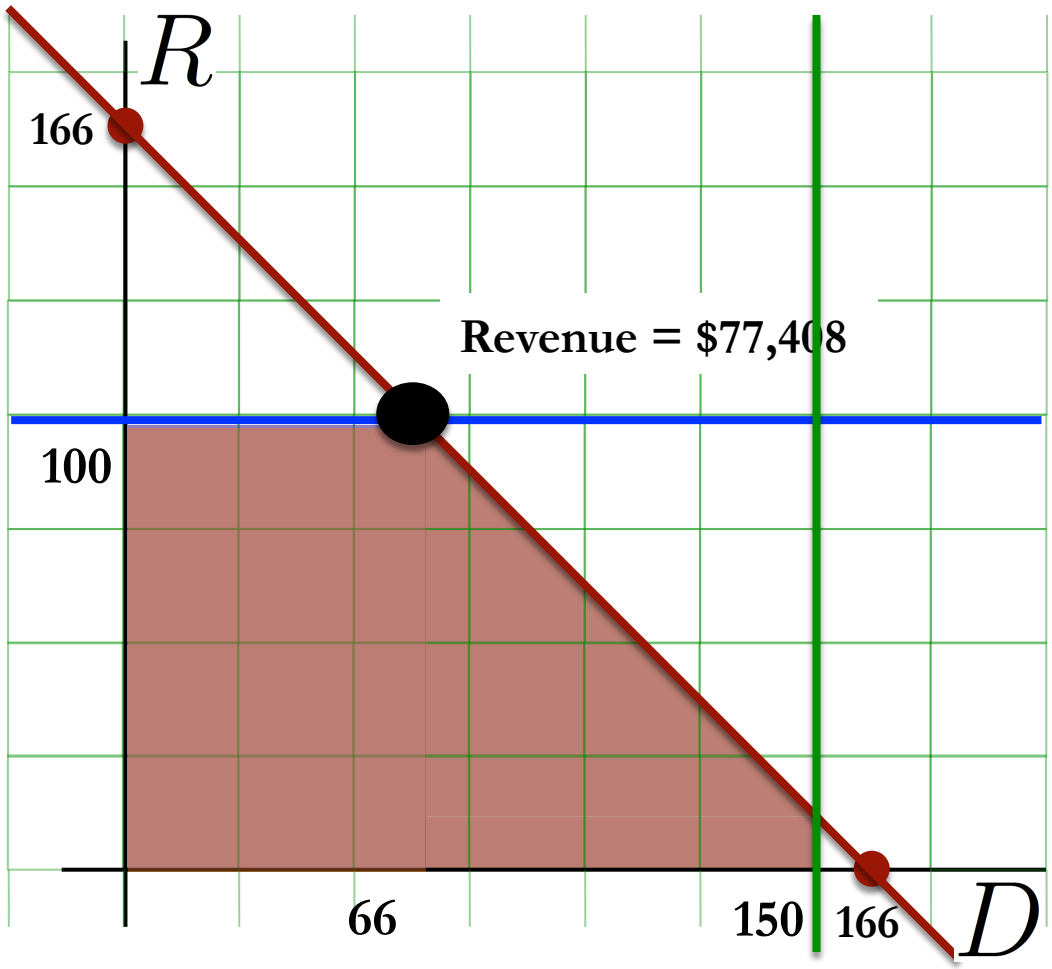
| | Marketing Cost/unit | Marginal Revenue |
|---|---|---|
| **Discount Fare** | $200 | |
| **Regular Fare** | $200 | |

# Marketing Discount Fares



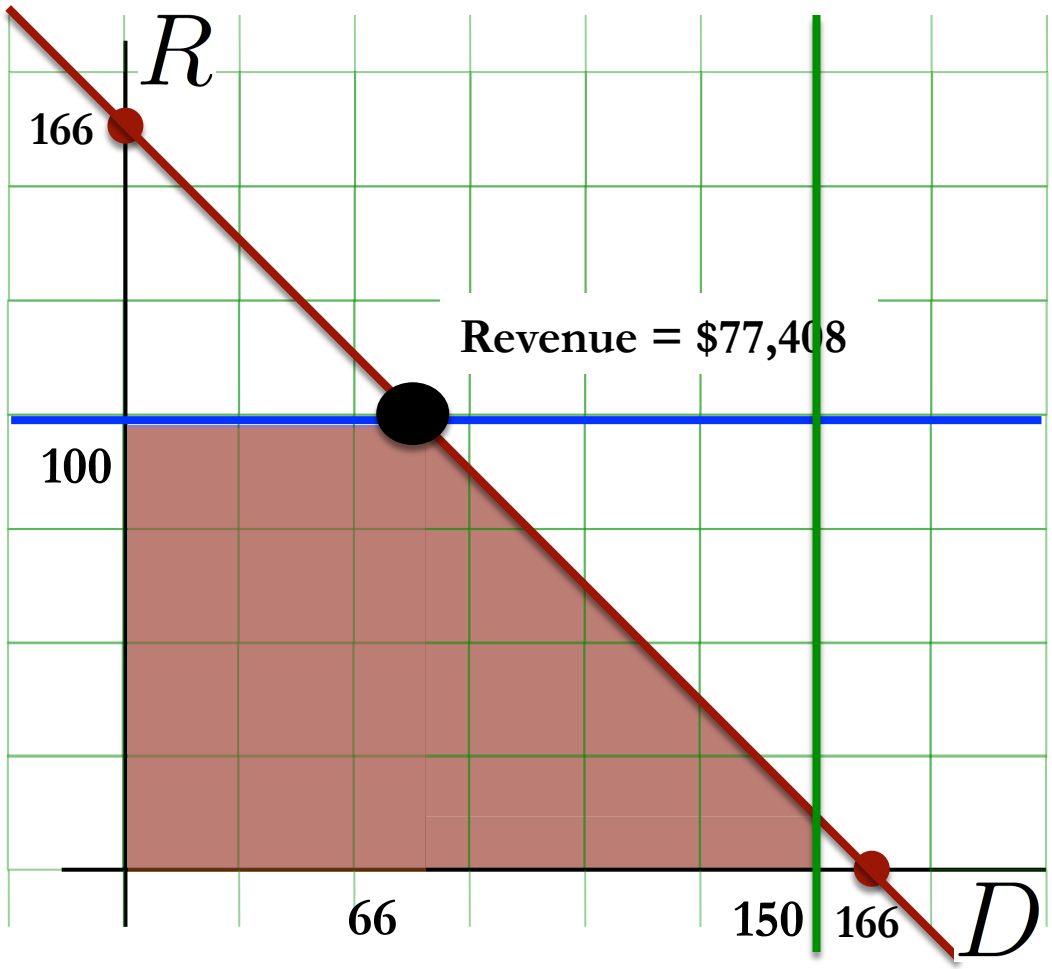

- What if AA increases its marketing budget for discount fares
- Higher demand for discount class
  - 150
  - 175
  - 200

# Marketing Discount Fares



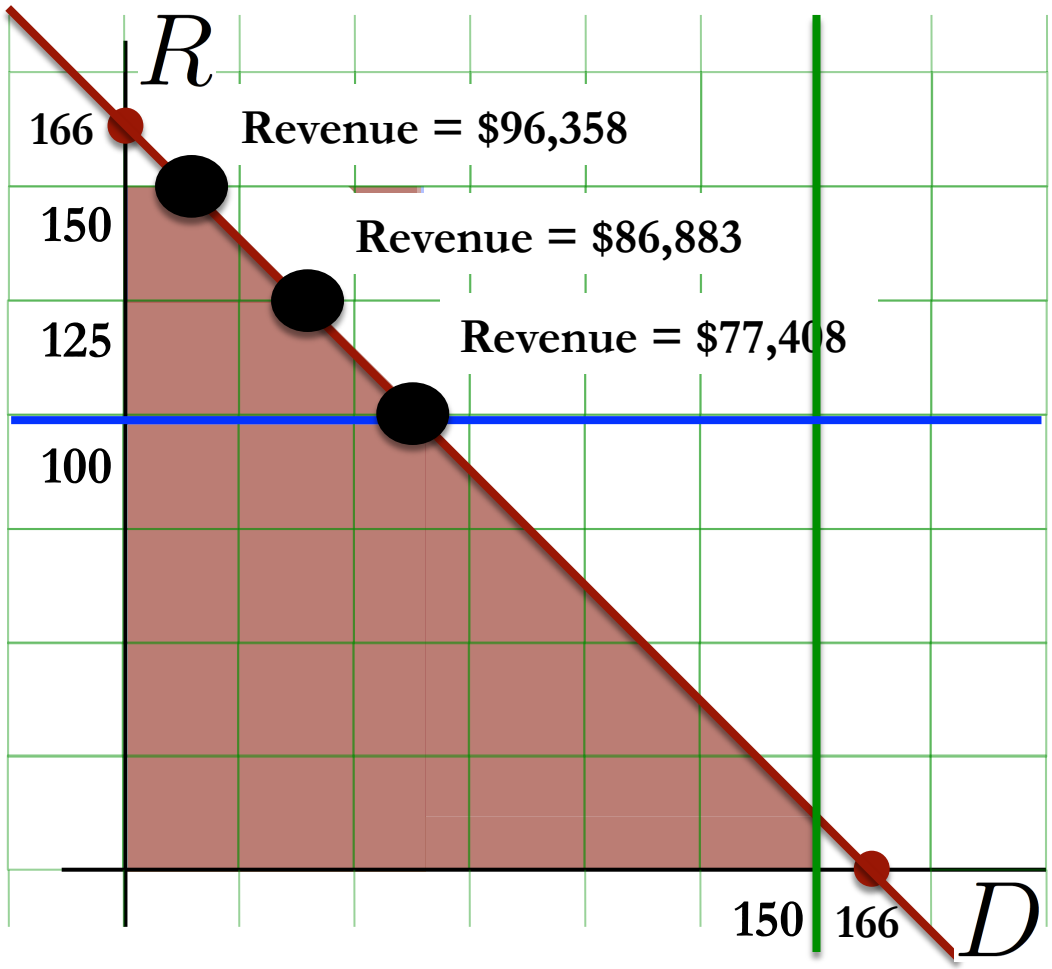

- What if AA decreases its budget to market discount fares?
- Lower demand for discount fare without affecting revenue

# Marketing Discount Fares



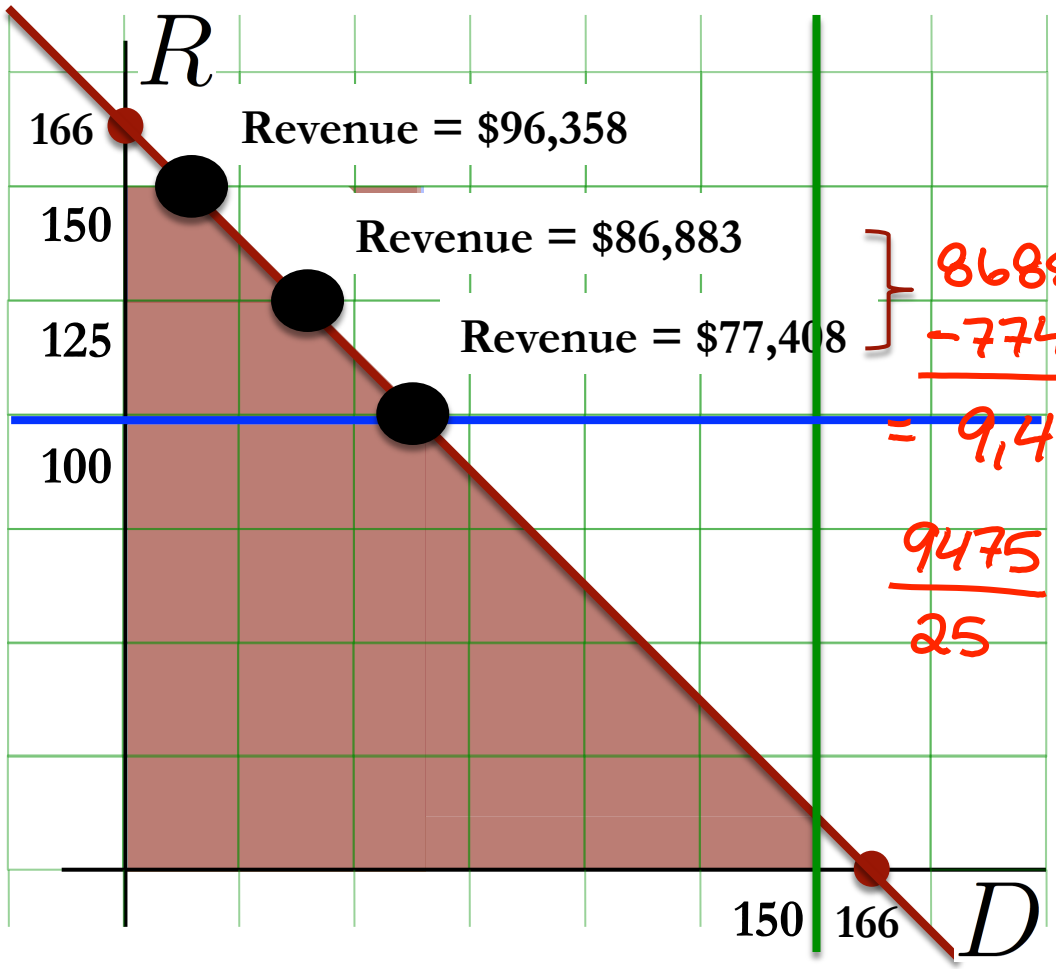

- "Shadow Price"
  - Marginal revenue of increasing discount demand by 1 unit
  - **ZERO** for discount demand greater than 66

# Marketing Regular Fares




- AA is considering increasing its budget to market regular fares
- Higher demand for regular class
  - 100
  - 125
  - 150

# Marketing Regular Fares




- "Shadow Price"
  - Marginal revenue for unit increase in demand of regular seats
  - **$379** for regular demand between 0 and 166

# Marketing Decisions

- Management is trying to figure out whether it would be beneficial to invest in marketing its fares

- AA forecasts that its marketing effort is likely to attract one more unit of demand per **$200 spent**

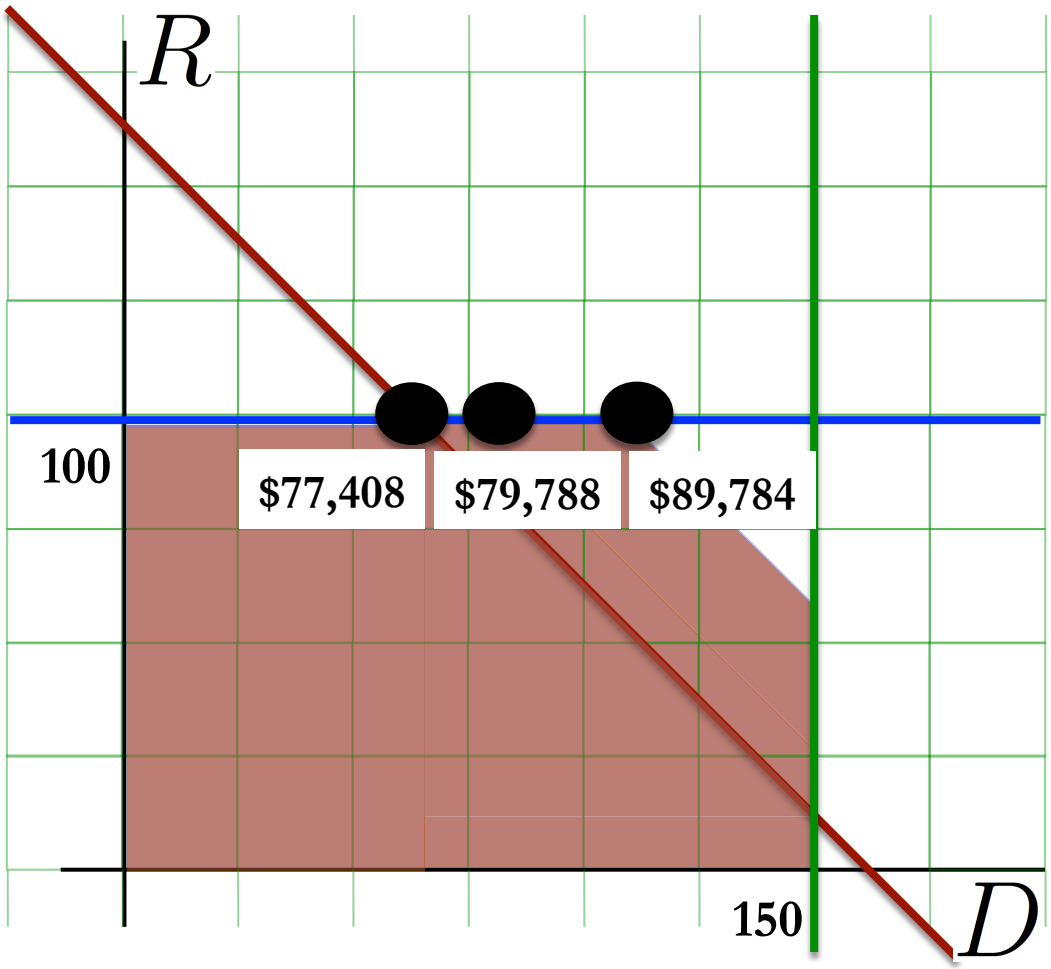
| | Marketing Cost/unit | Marginal Revenue |
|---|---|---|
| **Discount Fare** | $200 | 0 |
| **Regular Fare** | $200 | $379 |

# Capacity Allocation

- Management is trying to figure out whether it would be beneficial to allocate a bigger aircraft for the 6 hour JFK-LAX leg

| | Cost/hr | Total Cost | Seats | Revenue |
|---|---|---|---|---|
| **Original Aircraft** | $12,067 | $72,402 | 166 | $77,408 |
| **Boeing 757-200** | $12,765 | $76,590 | 176 | |
| **Boeing 767-300** | $14,557 | $87,342 | 218 | |

# Aircraft Capacity




- AA is considering increasing its aircraft capacity
  - 166
  - 176
  - 218

# Capacity Allocation

- Management is trying to figure out whether it would be beneficial to allocate a bigger aircraft for the 6 hour JFK-LAX leg

| | Total Cost | Revenue | Profit |
|---|---|---|---|
| **Original Aircraft** | $72,402 | $77,408 | $5,006 |
| **Boeing 757-200** | $76,590 | $79,788 | $3,198 |
| **Boeing 767-300** | $87,342 | $89,784 | $2,442 |

# Connecting Flights

# Step 1. Decisions

| | | Price | Demand | Seats to Sell | Flight Leg (capacity 166 on each) |
|---|---|---|---|---|---|
| JFK - LAX | Regular | 428 | 80 | ? | 1 & 2 |
| | Discount | 190 | 120 | ? | 1 & 2 |
| JFK - DFW | Regular | 642 | 75 | ? | 1 |
| | Discount | 224 | 100 | ? | 1 |
| DFW - LAX | Regular | 512 | 60 | ? | 2 |
| | Discount | 190 | 110 | ? | 2 |

- Number of regular seats to sell

$$R_{\text{JFK-LAX}}, R_{\text{JFK-DFW}}, R_{\text{DFW-LAX}}$$

- Number of discount seats to sell

$$D_{\text{JFK-LAX}}, D_{\text{JFK-DFW}}, D_{\text{DFW-LAX}}$$

# Step 2. Objective

| | | Price | Demand | Seats to Sell | Flight Leg (capacity 166 on each) |
|---|---|---|---|---|---|
| JFK - LAX | Regular | 428 | 80 | ? | 1 & 2 |
| | Discount | 190 | 120 | ? | 1 & 2 |
| JFK - DFW | Regular | 642 | 75 | ? | 1 |
| | Discount | 224 | 100 | ? | 1 |
| DFW - LAX | Regular | 512 | 60 | ? | 2 |
| | Discount | 190 | 110 | ? | 2 |

- Maximize total revenue

$$\begin{aligned} & 428R_{\text{JFK-LAX}} + 190D_{\text{JFK-LAX}} \\ & + 642R_{\text{JFK-DFW}} + 224D_{\text{JFK-DFW}} \\ & + 512R_{\text{DFW-LAX}} + 190D_{\text{DFW-LAX}} \end{aligned}$$

# Step 3. Constraints

| | | Price | Demand | Seats to Sell | Flight Leg (capacity 166 on each) |
|---|---|---|---|---|---|
| JFK - LAX | Regular | 428 | 80 | ? | 1 & 2 |
| | Discount | 190 | 120 | ? | 1 & 2 |
| JFK - DFW | Regular | 642 | 75 | ? | 1 |
| | Discount | 224 | 100 | ? | 1 |
| DFW - LAX | Regular | 512 | 60 | ? | 2 |
| | Discount | 190 | 110 | ? | 2 |

- AA cannot sell more seats that the aircraft capacity
  - First leg - JFK-DFW

$$R_{\text{JFK-LAX}} + D_{\text{JFK-LAX}} + R_{\text{JFK-DFW}} + D_{\text{JFK-DFW}} \leq 166$$

  - Second leg - DFW-LAX

$$R_{\text{JFK-LAX}} + D_{\text{JFK-LAX}} + R_{\text{DFW-LAX}} + D_{\text{DFW-LAX}} \leq 166$$

# Step 3. Constraints

| | | Price | Demand | Seats to Sell | Flight Leg (capacity 166 on each) |
|---|---|---|---|---|---|
| JFK - LAX | Regular | 428 | 80 | ? | 1 & 2 |
| | Discount | 190 | 120 | ? | 1 & 2 |
| JFK - DFW | Regular | 642 | 75 | ? | 1 |
| | Discount | 224 | 100 | ? | 1 |
| DFW - LAX | Regular | 512 | 60 | ? | 2 |
| | Discount | 190 | 110 | ? | 2 |

- AA cannot sell more seats than the demand

$$\begin{cases} R_{\text{JFK-LAX}} \leq 80 & D_{\text{JFK-LAX}} \leq 120 \\ R_{\text{JFK-DFW}} \leq 75 & D_{\text{JFK-DFW}} \leq 100 \\ R_{\text{DFW-LAX}} \leq 60 & D_{\text{DFW-LAX}} \leq 110 \end{cases}$$

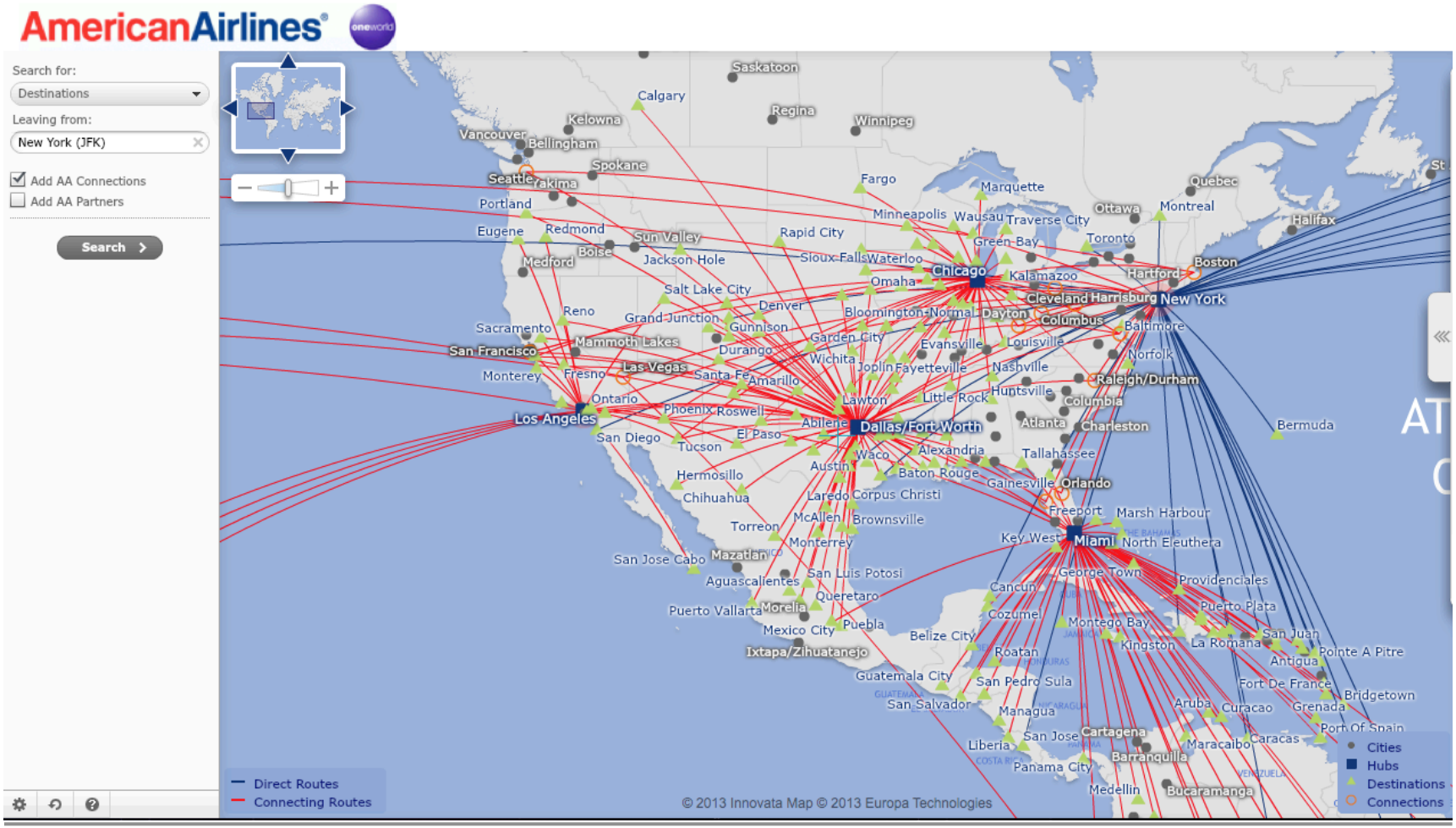
# Step 4. Non-Negativity

| | | Price | Demand | Seats to Sell | Flight Leg (capacity 166 on each) |
|---|---|---|---|---|---|
| JFK - LAX | Regular | 428 | 80 | ? | 1 & 2 |
| | Discount | 190 | 120 | ? | 1 & 2 |
| JFK - DFW | Regular | 642 | 75 | ? | 1 |
| | Discount | 224 | 100 | ? | 1 |
| DFW - LAX | Regular | 512 | 60 | ? | 2 |
| | Discount | 190 | 110 | ? | 2 |

- AA cannot sell a negative number of seats

$$\begin{cases} R_{\text{JFK-LAX}} \geq 0 & D_{\text{JFK-LAX}} \geq 0 \\ R_{\text{JFK-DFW}} \geq 0 & D_{\text{JFK-DFW}} \geq 0 \\ R_{\text{DFW-LAX}} \geq 0 & D_{\text{DFW-LAX}} \geq 0 \end{cases}$$

# Complex Network



# Multiple Fare Classes

| Fare | Domestic Upg. | International Upg. | EQP | EQM | Mileage |
|---|---|---|---|---|---|
| A | First Class | First Class | 1.5 | 1.0 | 150% |
| B | Yes | Yes | 1.5 | 1.0 | 100% |
| C | NA | Business Upgrade | N/A | N/A | N/A |
| D | NA | Business Fare | 1.5 | 1.0 | 125% |
| E | No | No | N/A | N/A | N/A |
| F | First Class Fare | First Class | 1.5 | 1.0 | 150% |
| G | Government | Government | .5 | 1.0 | 100% |
| H | Yes* | Waitlist only | 1.0 | 1.0 | 100% |
| I | NA | Business Class Fare | 1.5 | 1.0 | 125% |
| J | NA | Business Class Fare | 1.5 | 1.0 | 125% |
| K | Yes | No | 1.0 | 1.0 | 100% |
| L | Yes | No | 1.0 | 1.0 | 100% |
| M | Yes | No | 1.0 | 1.0 | 100% |

| Fare | Domestic Upg. | International Upg. | EQP | EQM | Mileage |
|---|---|---|---|---|---|
| N | Yes | No | .5 | 1.0 | 100% |
| O | Yes* | No | .5 | 1.0 | 100% |
| P | First Class Fare | First Class Fare | 1.5 | 1.0 | 150% |
| Q | Yes | No | .5 | 1.0 | 100% |
| R | NA | Business Class Upgrade or waitlist | N/A | N/A | N/A |
| S | Yes* | No | .5 | 1.0 | 100% |
| T | Coach Award | No | N/A | N/A | N/A |
| U | NA | Business Class Award | N/A | N/A | N/A |
| V | Yes* | No | 1.0 | 1.0 | 100% |
| W | Yes* | No | 1.0 | 1.0 | 100% |
| X | First Class Upgrade | Business Class Upgrade | N/A | N/A | N/A |
| Y | Yes | Yes | 1.5 | 1.0 | 100% |
| Z | First Class Award | NA | N/A | N/A | N/A |

**EQP: Elite-Qualifying Points / EQM: Elite-Qualifying Miles**

# The Competitive Strategy of AA

- PEOPLExpress could not compete with AA's Ultimate Super Savers fares

"We were a vibrant, profitable company from 1981 to 1985, and then we tipped right over into **losing 50 million a month**."

"We had been profitable from the day we started until American came at us with Ultimate Super Savers."

Donald Burr, CEO of PEOPLExpress (1985)

# The Competitive Strategy of AA

- Selling the right seats to the right customers at the right prices

"**Revenue management** is the single most important technical development in transportation management since we entered the era of airline deregulation."

"We estimate that revenue management has generated **$1.4 billion in incremental revenue** in the last three years."

Robert Crandall, former CEO of AA (~1985)

# The Edge of Revenue Management

- Sabre Holdings
  - Built revenue management system for AA
  - As of November 2012, ranked 133 among America's largest private companies with $3.15 billion in sales
  - 400 airlines, 90,000 hotels, 30 car-rental companies
- Today, companies prosper from revenue management
  - Delta airlines increased annual revenue by $300 million
  - Marriott hotels increased annual revenue by $100 million