

Week 0 - Overview of Course

0.1 Opening Remarks

- 0.1.1 Welcome
- 0.1.2 Outline Week 1
- 0.1.3 What You Will Learn

0.2 Navigating this Course

- 0.2.1 How to Navigate this Document
- 0.2.2 Setting Up Your Computer

0.3 Programming with MATLAB

- 0.3.1 Installing MATLAB
- 0.3.2 MATLAB Basics

0.4 Enrichments

- 0.4.1 The Origins of MATLAB

0.5 Wrap Up

- 0.5.1 Additional Homework
- 0.5.2 Summary

Week 1 - Vectors in Linear Algebra

1.1 Opening Remarks

- 1.1.1 Take Off
- 1.1.2 Outline Week 1
- 1.1.3 What You Will Learn

1.2 What is a Vector?

- 1.2.1 Notation
- 1.2.2 Unit Basis Vectors

1.3 Simple Vector Operations

- 1.3.1 Equality ($=$), Assignment ($:=$), and Copy
- 1.3.2 Vector Addition
- 1.3.3 Scaling
- 1.3.4 Vector Subtraction

1.4 Advanced Vector Operations

- 1.4.1 Scaled Vector Addition
- 1.4.2 Linear Combinations of Vectors
- 1.4.3 Dot or Inner Product
- 1.4.4 Vector Length
- 1.4.5 Vector Functions
- 1.4.6 Vector Functions that Map a Vector to a Vector

1.5 LAFF Package Development: Vectors

- 1.5.1 Starting the Package
- 1.5.2 A Copy Routine (copy)
- 1.5.3 A Routine that Scales a Vector
- 1.5.4 A Scaled Vector Addition Routine
- 1.5.5 An Inner Product Routine
- 1.5.6 A Vector Length Routine

1.6 Slicing and Dicing

- 1.6.1 Slicing and Dicing: Dot Product
- 1.6.2 Algorithms with Slicing and Redicing: Dot Product
- 1.6.3 Coding with Slicing and Redicing: Dot Product
- 1.6.4 Slicing and Dicing: axpy
- 1.6.5 Algorithms with Slicing and Redicing: axpy
- 1.6.6 Coding with Slicing and Redicing: axpy

1.7 Enrichment

- 1.7.1 Learn the Greek Alphabet
- 1.7.2 Other Norms
- 1.7.3 Overflow and Underflow
- 1.7.4 A Bit of History

1.8 Wrap Up

- 1.8.1 Homework
- 1.8.2 Summary of Vector Operations
- 1.8.3 Summary of the Properties of Vector Operations
- 1.8.4 Summary of the Routines for Vector Operations

Week 2 - Linear Transformations and Matrices

2.1 Opening Remarks

- 2.1.1 Rotating in 2D
- 2.1.2 Outline
- 2.1.3 What You Will Learn

2.2 Linear Transformations

- 2.2.1 What Makes Linear Transformations so Special?
- 2.2.2 What is a Linear Transformation?
- 2.2.3 Of Linear Transformations and Linear Combinations

2.3 Mathematical Induction

- 2.3.1 What is the Principle of Mathematical Induction?
- 2.3.2 Examples

2.4 Representing Linear Transformations as Matrices

- 2.4.1 From Linear Transformation to Matrix-Vector Multiplication
- 2.4.2 Practice with Matrix-Vector Multiplication
- 2.4.3 It Goes Both Ways
- 2.4.4 Rotations and Reflections, Revisited

2.5 Enrichment

- 2.5.1 The Importance of the Principle of Mathematical Induction for Programming
- 2.5.2 Puzzles and Paradoxes in Mathematical Induction

2.6 Wrap Up

- 2.6.1 Homework
- 2.6.2 Summary

Week 3 - Matrix-Vector Operations

3.1 Opening Remarks

- 3.1.1 Timmy Two Space
- 3.1.2 Outline Week 3
- 3.1.3 What You Will Learn

3.2 Special Matrices

- 3.2.1 The Zero Matrix
- 3.2.2 The Identity Matrix
- 3.2.3 Diagonal Matrices
- 3.2.4 Triangular Matrices
- 3.2.5 Transpose Matrix
- 3.2.6 Symmetric Matrices

3.3 Operations with Matrices

- 3.3.1 Scaling a Matrix
- 3.3.2 Adding Matrices

3.4 Matrix-Vector Multiplication Algorithms

- 3.4.1 Via Dot Products
- 3.4.2 Via $axpy$
- 3.4.3 Compare and Contrast
- 3.4.4 Cost of Matrix-Vector Multiplication

3.5 Wrap Up

- 3.5.1 Homework
- 3.5.2 Summary

Week 4 - From Matrix-Vector Multiplication to Matrix-Matrix Multiplication

4.1 Opening Remarks

- 4.1.1 Predicting the Weather
- 4.1.2 Outline
- 4.1.3 What You Will Learn

4.2 Preparation

- 4.2.1 Partitioned Matrix-Vector Multiplication
- 4.2.2 Transposing a Partitioned Matrix
- 4.2.3 Matrix-Vector Multiplication, Again

4.3 Matrix-Vector Multiplication with Special Matrices

- 4.3.1 Transpose Matrix-Vector Multiplication
- 4.3.2 Triangular Matrix-Vector Multiplication
- 4.3.3 Symmetric Matrix-Vector Multiplication

4.4 Matrix-Matrix Multiplication (Product)

- 4.4.1 Motivation
- 4.4.2 From Composing Linear Transformations to Matrix-Matrix Multiplication
- 4.4.3 Computing the Matrix-Matrix Product
- 4.4.4 Special Shapes
- 4.4.5 Cost

4.5 Enrichment

- 4.5.1 Hidden Markov Processes

4.6 Wrap Up

4.6.1 Homework

4.6.2 Summary

Week 5 - Matrix-Matrix Multiplication

5.1 Opening Remarks

5.1.1 Composing Rotations

5.1.2 Outline

5.1.3 What You Will Learn

5.2 Observations

5.2.1 Partitioned Matrix-Matrix Multiplication

5.2.2 Properties

5.2.3 Transposing a Product of Matrices

5.2.4 Matrix-Matrix Multiplication with Special Matrices

5.3 Algorithms for Computing Matrix-Matrix Multiplication

5.3.1 Lots of Loops

5.3.2 Matrix-Matrix Multiplication by Columns

5.3.3 Matrix-Matrix Multiplication by Rows

5.3.4 Matrix-Matrix Multiplication with Rank-1 Updates

5.4 Enrichment

5.4.1 Slicing and Dicing for Performance

5.4.2 How It is Really Done

5.5 Wrap Up

5.5.1 Homework

5.5.2 Summary

Week 6 - Gaussian Elimination

6.1 Opening Remarks

6.1.1 Solving Linear Systems

6.1.2 Outline

6.1.3 What You Will Learn

6.2 Gaussian Elimination

6.2.1 Reducing a System of Linear Equations to an Upper Triangular System

6.2.2 Appended Matrices

6.2.3 Gauss Transforms

6.2.4 Computing Separately with the Matrix and Right-Hand Side (Forward Substitution)

6.2.5 Towards an Algorithm

6.3 Solving $Ax = b$ via LU Factorization

6.3.1 LU factorization (Gaussian elimination)

6.3.2 Solving $Lz = b$ (Forward substitution)

6.3.3 Solving $Ux = b$ (Back substitution)

6.3.4 Putting it all together to solve $Ax = b$

6.3.5 Cost

6.4 Enrichment

6.4.1 Blocked LU Factorization

6.5 Wrap Up

6.5.1 Homework

6.5.2 Summary

Week 7 - More Gaussian Elimination and Matrix Inversion

7.1 Opening Remarks

7.1.1 Introduction

7.1.2 Outline

7.1.3 What You Will Learn

7.2 When Gaussian Elimination Breaks Down

7.2.1 When Gaussian Elimination Works

7.2.2 The Problem

7.2.3 Permutations

7.2.4 Gaussian Elimination with Row Swapping (LU Factorization with Partial Pivoting)

7.2.5 When Gaussian Elimination Fails Altogether

7.3 The Inverse Matrix

7.3.1 Inverse Functions in 1D

7.3.2 Back to Linear Transformations

7.3.3 Simple Examples

7.3.4 More Advanced (but Still Simple) Examples

7.3.5 Properties

7.4 Enrichment

7.4.1 Library Routines for LU with Partial Pivoting

7.5 Wrap Up

7.5.1 Homework

7.5.2 Summary

Week 8 - More on Matrix Inversion

8.1 Opening Remarks

8.1.1 When LU Factorization with Row Pivoting Fails

8.1.2 Outline

8.1.3 What You Will Learn

8.2 Gauss-Jordan Elimination

8.2.1 Solving $Ax = b$ via Gauss-Jordan Elimination

8.2.2 Solving $Ax = b$ via Gauss-Jordan Elimination: Gauss Transforms

8.2.3 Solving $Ax = b$ via Gauss-Jordan Elimination: Multiple Right-Hand Sides

8.2.4 Computing A^{-1} via Gauss-Jordan Elimination

8.2.5 Computing A^{-1} via Gauss-Jordan Elimination, Alternative

8.2.6 Pivoting

8.2.7 Cost of Matrix Inversion

8.3 (Almost) Never, Ever Invert a Matrix

8.3.1 Solving $Ax = b$

8.3.2 But...

8.4 (Very Important) Enrichment

8.4.1 Symmetric Positive Definite Matrices

8.4.2 Solving $Ax = b$ when A is Symmetric Positive Definite

8.4.3 Other Factorizations

8.4.4 Welcome to the Frontier

8.5 Wrap Up

8.5.1 Homework

8.5.2 Summary

Week 9 - Vector Spaces

9.1 Opening Remarks

9.1.1 Solvable or not solvable, that's the question

9.1.2 Outline

9.1.3 What you will learn

9.2 When Systems Don't Have a Unique Solution

9.2.1 When Solutions Are Not Unique

9.2.2 When Linear Systems Have No Solutions

9.2.3 When Linear Systems Have Many Solutions

9.2.4 What is Going On?

9.2.5 Toward a Systematic Approach to Finding All Solutions

9.3 Review of Sets

9.3.1 Definition and Notation

9.3.2 Examples

9.3.3 Operations with Sets

9.4 Vector Spaces

9.4.1 What is a Vector Space?

9.4.2 Subspaces

9.4.3 The Column Space

9.4.4 The Null Space

9.5 Span, Linear Independence, and Bases

9.5.1 Span

9.5.2 Linear Independence

9.5.3 Bases for Subspaces

9.5.4 The Dimension of a Subspace

9.6 Enrichment

9.6.1 Typesetting algorithms with the FLAME notation

9.7 Wrap Up

9.7.1 Homework

9.7.2 Summary

Week 10 - Vector Spaces, Orthogonality, and Linear Least Squares

10.1 Opening Remarks

- 10.1.1 Visualizing Planes, Lines, and Solutions
- 10.1.2 Outline
- 10.1.3 What You Will Learn

10.2 How the Row Echelon Form Answers (Almost) Everything

- 10.2.1 Example
- 10.2.2 The Important Attributes of a Linear System

10.3 Orthogonal Vectors and Spaces

- 10.3.1 Orthogonal Vectors
- 10.3.2 Orthogonal Spaces
- 10.3.3 Fundamental Spaces

10.4 Approximating a Solution

- 10.4.1 A Motivating Example
- 10.4.2 Finding the Best Solution
- 10.4.3 Why It is Called Linear Least-Squares

10.5 Enrichment

- 10.5.1 Solving the Normal Equations

10.6 Wrap Up

- 10.6.1 Homework
- 10.6.2 Summary

Week 11 - Orthogonal Projection, Low Rank Approximation, and Orthogonal Bases

11.1 Opening Remarks

- 11.1.1 Low Rank Approximation
- 11.1.2 Outline
- 11.1.3 What You Will Learn

11.2 Projecting a Vector onto a Subspace

- 11.2.1 Component in the Direction of ...
- 11.2.2 An Application: Rank-1 Approximation
- 11.2.3 Projection onto a Subspace
- 11.2.4 An Application: Rank-2 Approximation
- 11.2.5 An Application: Rank-k Approximation

11.3 Orthonormal Bases

- 11.3.1 The Unit Basis Vectors, Again
- 11.3.2 Orthonormal Vectors
- 11.3.3 Orthogonal Bases
- 11.3.4 Orthogonal Bases (Alternative Explanation)
- 11.3.5 The QR Factorization
- 11.3.6 Solving the Linear Least-Squares Problem via QR Factorization
- 11.3.7 The QR Factorization (Again)

11.4 Orthonormal Bases

- 11.4.1 The Unit Basis Vectors, One More Time
- 11.4.2 Change of Basis

11.5 Singular Value Decomposition

11.5.1 The Best Low Rank Approximation

11.6 Enrichment

11.6.1 The Problem with Computing the QR Factorization

11.6.2 QR Factorization Via Householder Transformations
(Reflections)

11.6.3 More on SVD

11.7 Wrap Up

11.7.1 Homework

11.7.2 Summary

Week 12 - Eigenvalues, Eigenvectors, and Diagonalization

12.1 Opening Remarks

12.1.1 Predicting the Weather, Again

12.1.2 Outline

12.1.3 What You Will Learn

12.2 Getting Started

12.2.1 The Algebraic Eigenvalue Problem

12.2.2 Simple Examples

12.2.3 Diagonalizing

12.2.4 Eigenvalues and Eigenvectors of 3×3 Matrices

12.3 The General Case

12.3.1 Eigenvalues and Eigenvectors of $n \times n$ matrices:
Special Cases

12.3.2 Eigenvalues of $n \times n$ Matrices

12.3.3 Diagonalizing, Again

12.3.4 Properties of Eigenvalues and Eigenvectors

12.4 Practical Methods for Computing Eigenvectors and Eigenvalues

12.4.1 Predicting the Weather, One Last Time

12.4.2 The Power Method

12.4.3 In Preparation for this Week's Enrichment

12.5 Enrichment

12.5.1 The Inverse Power Method

12.5.2 The Rayleigh Quotient Iteration

12.5.3 More Advanced Techniques

12.6 Wrap Up

12.6.1 Homework

12.6.2 Summary